

COMPETENCY-BASED QUESTION BANK

**WITH ANSWER KEY
& STRUCTURED EXPLANATION**

CLASS 12
MATHEMATICS



FEATURES

1 Comprehensive Coverage

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MATRICES

1. If n is a natural number. Then $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}^n$, is
 - a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is even
 - b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is odd
 - c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is a natural number
 - d) None of these
2. $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = I$, then
 - a) $a = 1 = 2b$
 - b) $a = b$
 - c) $a = b^2$
 - d) $ab = 1$
3. If a square matrix A is such that $AA^T = I = A^T A$, then $|A|$ is equal to
 - a) 0
 - b) ± 1
 - c) ± 2
 - d) None of these
4. If k is a scalar and I is a unit matrix of order 3, then $\text{adj}(kI)$ is equal to
 - a) $k^3 I$
 - b) $k^2 I$
 - c) $-k^3 I$
 - d) $-k^2 I$
5. If A is a singular matrix, then $A \text{adj}(A)$ is a
 - a) Scalar matrix
 - b) Zero matrix
 - c) Identity matrix
 - d) Orthogonal matrix
6. If $A = \begin{bmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{bmatrix}$ are two matrices such that the product AB is null matrix, then $\alpha - \beta$ is
 - a) 0
 - b) Multiple of π
 - c) An odd multiple of $\pi/2$
 - d) None of the above
7. Consider the following statements:
 1. If A and B are two square matrices of same order and commute, then $(A + B)(A - B) = A^2 - B^2$
 2. If A and B are two square matrices of same order, then $(AB)^n = A^n B^n$
 3. If A and B are two matrices such that $AB = A$ and $BA = B$, then A and B are idempotent
 Which of these is/are not correct?
 - a) Only (1)
 - b) (2) and (3)
 - c) (3) and (1)
 - d) All of these
8. If $A = \begin{bmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ -6 & -12 & -15 \end{bmatrix}$, the rank of A is equal to
 - a) 0
 - b) 1
 - c) 2
 - d) 3
9. If A and B are 3×3 matrices such that $AB + B = A$ and $BA = A$, then
 - a) $A^2 = A$ and $B^2 \neq B$
 - b) $A^2 \neq A$ and $B^2 = B$
 - c) $A^2 = A$ and $B^2 = B$
 - d) $A^2 \neq A$ and $B^2 \neq B$
10. The inverse of the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is
 - a) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix}$
 - b) $\begin{bmatrix} 1 & 3 & 1 \\ 4 & 3 & 8 \\ 3 & 4 & 1 \end{bmatrix}$
 - c) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 3 & 4 & 3 \end{bmatrix}$
 - d) $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$
11. If A is a skew-symmetric matrix and n is a positive integer, then A^n is
 - a) A symmetric matrix
 - b) Skew-symmetric matrix
 - c) Diagonal matrix
 - d) None of these
12. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is
 - a) Less than 4
 - b) 5
 - c) 6
 - d) At least 7
13. If $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$, then trace of matrix A is
 - a) 17
 - b) 25
 - c) 3
 - d) 12

14. If $A = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix}$, then $(A + B)^{-1}$ is equal to
- a) Is a skew-symmetric matrix
b) $A^{-1} + B^{-1}$
c) Does not exist
d) None of the above
15. If $f(x) = x^2 - 5x$, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $f(A)$ is equal to
- a) $\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$
b) $\begin{bmatrix} 0 & -7 \\ -7 & 0 \end{bmatrix}$
c) $\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$
d) $\begin{bmatrix} 0 & 7 \\ 7 & 0 \end{bmatrix}$
16. The adjoint matrix of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ is
- a) $\begin{bmatrix} 4 & 8 & 3 \\ 2 & 1 & 6 \\ 0 & 2 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$
c) $\begin{bmatrix} 11 & 9 & 3 \\ 1 & 2 & 8 \\ 6 & 9 & 1 \end{bmatrix}$
d) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 3 \\ -2 & 3 & -3 \end{bmatrix}$
17. If the rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is 1, then the value of a is
- a) -1
b) 2
c) -6
d) 4
18. The value of λ such that $x + 3y + \lambda z = 0$, $2x + 4y - z = 0$, $x + 5y - 2z = 0$ has a non-trivial solution is
- a) -1
b) 0
c) 1
d) 2
19. If $A = [a_{ij}]_{m \times n}$ is a matrix and B is a non-singular square submatrix of order r , then
- a) Rank of A is r
b) Rank of A is greater than r
c) Rank of A is less than r
d) None of these
20. From the matrix equation $AB = AC$ we can conclude $B = C$ provided that
- a) A is singular
b) A is non-singular
c) A is symmetric
d) A is square
21. The values of α for which the system of equations
- $$\begin{aligned} x + y + z &= 1 \\ x + 2y + 4z &= \alpha \\ x + 4y + 10z &= \alpha^2 \end{aligned}$$
- Is consistent, are
- a) 1, -2
b) -1, 2
c) 1, 2
d) None of these
22. If A is a non-singular matrix such that $A^3 = A + I$, then the inverse of $B = A^6 - A^5$ is
- a) A
b) A^{-1}
c) $-A$
d) $-A^{-1}$
23. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $A^T A = I$, then the value of $a^3 + b^3 + c^3$ is
- a) 1
b) 2
c) 3
d) 4
24. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then $A^2 - 4A$ is equal to
- a) $2I_3$
b) $3I_3$
c) $4I_3$
d) $5I_3$
25. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then $A^4 =$
- a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$
c) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$
d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
26. The simultaneous equations $Kx + 2y - z = 1$, $(K - 1)y - 2z = 2$ and $(K + 2)z = 3$ have only one solution when
- a) $K = -2$
b) $K = -1$
c) $K = 0$
d) $K = 1$
27. The system of equation,
- $$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \end{aligned}$$

And $x + 2y + \lambda z = \mu$

Has no solution, if

- a) $\lambda = 3, \mu = 10$ b) $\lambda \neq 3, \mu = 10$ c) $\lambda \neq 3, \mu \neq 10$ d) $\lambda = 3, \mu \neq 10$

28. If a square matrix A is such that $AA^T = I = A^T A$, then $|A|$ is equal to

- a) 0 b) ± 1 c) ± 2 d) None of these

29. Let $A = \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}$, then $AB = O$, if

- a) $\theta = n\phi, n = 0, 1, 2, \dots$ b) $\theta + \phi = n\pi, n = 0, 1, 2, \dots$ c) $\theta = \phi + (2n + 1)\frac{\pi}{2}, n = 0, 1, 2, \dots$ d) $\theta = \phi + n\frac{\pi}{2}, n = 0, 1, 2, \dots$

30. If A, B are two square matrices such that $AB = A$ and $BA = B$, then

- a) A, B are idempotent b) Only A is idempotent c) Only B is idempotent d) None of these

31. If $\begin{bmatrix} x - y - z \\ -y + z \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$, then the values of x, y and z are respectively

- a) 5, 2, 2 b) 1, -2, 3 c) 0, -3, 3 d) 11, 8, 3

32. Inverse of the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ is

- a) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ b) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$ c) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ d) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

33. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ and B is the inverse of A , then the value of α is

- a) 2 b) 0 c) 5 d) 4

34. $A = \begin{bmatrix} 0 & 3 & 3 \\ -3 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, then $B'(AB)$ is

- a) Null matrix b) Singular matrix c) Unit matrix d) Symmetric matrix

35. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then A^{-1} is

- a) $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$

- b) $\begin{bmatrix} -1/a & 0 & 0 \\ 0 & -1/b & 0 \\ 0 & 0 & -1/c \end{bmatrix}$

- c) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$

- d) None of these

36. If $A = \begin{bmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $a^2 + b^2 = 1$, then $\text{adj}(A)$ is equal to

(Here, A^T is the transpose of A)

- a) A^{-1} b) A^T c) A d) $-A$

37. The rank of a null matrix is

- a) 0 b) 1 c) Does not exist d) None of these

38. If A and B are matrices of the same order, then $(A + B)^2 = A^2 + 2AB + B^2$ is possible, iff

- a) $AB = I$ b) $BA = I$ c) $AB = BA$ d) None of these

39. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then x equals

- a) 2 b) $-\frac{1}{2}$ c) 1 d) $\frac{1}{2}$

40. If the system of equations

$$ax + ay - z = 0$$

$$bx - y + bz = 0$$

$$\text{and } -x + cy + cz = 0$$

Has a non-trivial solution, then the value of

$$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} \text{ is}$$

- a) 0 b) 1 c) 2 d) 3

41. The system of equation

$$3x - y + 4z = 3$$

$$x + 2y - 3z = -2$$

$$6x + 5y + \lambda z = -3$$

Has at least one solution, if

- a) $\lambda = -5$ b) $\lambda = 5$ c) $\lambda = 3$ d) $\lambda = -13$

42. If A is a skew-symmetric matrix and n is an even positive integer, then A^n is

- a) A symmetric matrix
b) A skew-symmetric matrix
c) A diagonal matrix
d) None of these

43. If $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then $|\text{adj } A|$ is equal to

- a) 0 b) 9 c) $\frac{1}{9}$ d) 81

44. If A is skew-symmetric matrix of order 3, then matrix A^3 is

- a) Skew-symmetric matrix b) Symmetric matrix
c) Diagonal matrix d) None of the above

45. If $[1 \ x \ 1] \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$, then the value of x is

- a) 0 b) $\frac{2}{3}$ c) $\frac{5}{4}$ d) $-\frac{4}{5}$

46. If $A = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}$, then $(A + B)^{-1} =$

- a) Is a skew-symmetric matrix
b) $A^{-1} + B^{-1}$
c) Does not exist
d) None of these

47. Let A be an orthogonal non-singular matrix of order n , then the determinant of matrix ' $A - I'_n$ ie, $|A - I_n|$ is equal to

- a) $|I_n - A|$ b) $|A|$ c) $|A||I_n - A|$ d) $(-1)^n |A||I_n - A|$

48. The matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ is

- a) Unitary b) Orthogonal c) Nilpotent d) Involutory

49. The inverse of a symmetric matrix is

- a) Symmetric b) Skew-symmetric c) Diagonal matrix d) None of these

50. The characteristic roots of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ are

- a) 1, 3, 6 b) 1, 2, 4 c) 4, 5, 6 d) 2, 4, 6

51. If matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{k} \text{adj } A$, then k is

a) 7

b) -7

c) $\frac{1}{7}$

d) 11

52. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has

exactly two distinct solutions, is

a) 0

b) $2^9 - 1$

c) 168

d) 2

53. If A is an invertible matrix of order n , then the determinant of $\text{adj}(A)$ is equal to

a) $|A|^n$ b) $|A|^{n+1}$ c) $|A|^{n-1}$ d) $|A|^{n+2}$

54. $\text{adj} \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix}$, then $[a \ b]$ is equal to

a) $[-4 \ 1]$ b) $[-4 \ -1]$ c) $[4 \ 1]$ d) $[4 \ -1]$

55. If $A = [x \ y \ z]$, $B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ and $C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ Then, $B'(AB) = O$, if

a) $[ax^2 + by^2 + cz^2 + 2gxy + 2fyz + 2czz] = O$ b) $[ax^2 + cy^2 + bz^2 + xy + yz + zx] = O$ c) $[ax^2 + by^2 + cz^2 + 2hxy + 2by + 2cz] = O$ d) $[ax^2 + by^2 + cz^2 + 2gzx + 2hxy + 2fyz] = O$

56. The multiplicative inverse of matrix $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ is

a) $\begin{bmatrix} 4 & -1 \\ -7 & -2 \end{bmatrix}$ b) $\begin{bmatrix} -4 & -1 \\ 7 & -2 \end{bmatrix}$ c) $\begin{bmatrix} 4 & -7 \\ 7 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

57. The rank of matrix $\begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 6 & 0 & 2 & 0 \end{bmatrix}$ is

a) 4

b) 3

c) 2

d) 1

58. The solution of (x, y, z) the equation

$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ is (x, y, z)

a) $(1, 1, 1)$ b) $(0, -1, 2)$ c) $(-1, 2, 2)$ d) $(-1, 0, 2)$

59. $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is equal to

a) $\begin{bmatrix} 43 \\ 44 \end{bmatrix}$ b) $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$ c) $\begin{bmatrix} 45 \\ 44 \end{bmatrix}$ d) $\begin{bmatrix} 44 \\ 45 \end{bmatrix}$

60. Let A, B and C be $n \times n$ matrices. Which one of the following is a correct statements?

a) If $AB = AC$, then $B = C$ b) If $A^3 + 2A^2 + 3A + 5I = O$; then A is invertiblec) If $A^2 = O$, then $A = O$

d) None of the above

61. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then $|3AB|$ equals

a) -9

b) -81

c) -27

d) 81

62. The rank of the matrix $\begin{bmatrix} 1230 \\ 2432 \\ 3213 \\ 6875 \end{bmatrix}$ is

a) 1

b) 2

c) 3

d) 4

63. If $A + 1 = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, then $(A - I)$ is equal to

a) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -5 & 4 \\ -8 & 9 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 4 \\ 8 & 9 \end{bmatrix}$ d) $\begin{bmatrix} -5 & -4 \\ -8 & -9 \end{bmatrix}$

64. If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ii} = k$ for all i , then $|A| =$

a) nk b) $n + k$ c) n^k d) k^n

65. If $A \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is such that $|A| = 0$ and $A^2 - (a - d)A + kI = 0$, then k is equal to

a) $b + c$ b) $a + d$ c) $ab + cd$

d) Zero

66. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $(AB)^T$ is equal to
a) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ b) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$ c) $\begin{bmatrix} -3 & 7 \\ 10 & 2 \end{bmatrix}$ d) None of these
67. If A is a matrix such that there exists a square submatrix of order r which is non-singular and every square submatrix of order $r + 1$ or more is singular, then
a) $\text{rank}(A) = r + 1$ b) $\text{rank}(A) = r$ c) $\text{rank}(A) > r$ d) $\text{rank}(A) \geq r + 1$
68. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then $A^2 - 5A =$
a) I b) $14I$ c) 0 d) None of these
69. The order of $[x \ y \ z] \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is
a) 3×1 b) 1×1 c) 1×3 d) 3×3
70. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then $A =$
a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$ c) $\begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$ d) None of these
71. If A is singular matrix, then $A \text{ adj}(A)$
a) Is a scalar matrix b) Is a zero matrix
c) Is an identity matrix d) Is an orthogonal matrix
72. If $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$, then X is equal to
a) $\begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$ b) $\begin{bmatrix} 3 & -4 \\ -14 & 13 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 4 \\ 14 & 13 \end{bmatrix}$ d) $\begin{bmatrix} -3 & 4 \\ -14 & 13 \end{bmatrix}$
73. For the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$, which of the following is correct?
a) $A^3 + 3A^2 - I = 0$ b) $A^3 - 3A^2 - I = 0$ c) $A^3 + 2A^2 - I = 0$ d) $A^3 - A^2 + I = 0$
74. The matrices $P = \begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix}$; $Q = \frac{1}{9} \begin{bmatrix} 2 & 2 & 1 \\ 12 & -5 & m \\ -8 & 1 & 5 \end{bmatrix}$ are such that $PQ = I$, an identity matrix. Solving the equation $\begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$, the value of y comes out to be -3 . Then, the value of m is equal to
a) 27 b) 7 c) -27 d) -7
75. If A is an invertible matrix, then which of the following is correct
a) A^{-1} is multivalued b) A^{-1} is singular c) $(A^{-1})^T \neq (A^T)^{-1}$ d) $|A| \neq 0$
76. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the matrix A equals
a) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$
77. If A is any square matrix, then $\det(A - A^T)^T$ is equal to
a) 0 b) 1
c) Can be 0 or a perfect square d) Cannot be determined
78. If $O(A) = 2 \times 3$, $O(B) = 3 \times 2$, and $O(C) = 3 \times 3$, Which one of the following is not defined?
a) $CB + A'$ b) BAC c) $C(A + B)'$ d) $C(A + B')$
79. Suppose A is a matrix of order 3 and $B = |A|A^{-1}$. If $|A| = -5$, then $|B|$ is equal to
a) 1 b) -5 c) -1 d) 25
80. $A = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$, then A^2 is equal to
a) Null matrix b) Unit matrix c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
81. The system of simultaneous equations $kx + 2y - z = 1$ $(k - 1)y - 2z = 2$ and $(k + 2)z = 3$ has a unique solution, if k equals

- a) -2 b) -1 c) 0 d) 1

82. If $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $Ax = B$, then X is equal to
a) $\begin{bmatrix} 0 & 7 \end{bmatrix}$ b) $\frac{1}{3} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$ c) $\frac{1}{3} \begin{bmatrix} 5 & 7 \end{bmatrix}$ d) $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

83. For the equations $x + 2y + 3z = 1$, $2x + y + 3z = 2$ and $5x + 5y + 9z = 4$
a) There is only one solution b) There exists infinitely many solutions
c) There is no solution d) None of the above

84. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $n \in N$, then A^n is equal to
a) $2^n A$ b) $2^{n-1} A$ c) $n A$ d) None of these

85. If $A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then $(B^{-1}A^{-1})^{-1} =$
a) $\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 3 & -2 \\ 2 & 2 \end{bmatrix}$ c) $\frac{1}{10} \begin{bmatrix} 2 & 2 \\ -2 & 3 \end{bmatrix}$ d) $\frac{1}{10} \begin{bmatrix} 3 & 2 \\ -2 & 2 \end{bmatrix}$

86. If A is a skew symmetric matrix of order n and C is a column matrix of order $n \times 1$, then $C^T A C$ is
a) An identity matrix of order n b) An identity matrix of order 1
c) A zero matrix of order 1 d) None of the above

87. If A is a square matrix, then $A + A^T$ is
a) Non-singular matrix b) Symmetric matrix
c) Skew-symmetric matrix d) Unit matrix

88. If $A = [a_{ij}]_{2 \times 2}$, where $a_{ij} = i + j$, then A is equal to
a) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

89. If $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$, then P_{22} is equal to
a) 40 b) -40 c) -20 d) 20

90. Which of the following is/are incorrect?
(i) adjoint of a symmetric matrix is symmetric
(ii) adjoint of a unit matrix is a unit matrix
(iii) $A(\text{adj } A) = (\text{adj } A)A = |A| I$
(iv) adjoint of a diagonal matrix is a diagonal matrix
a) (i) b) (ii) c) (iii) and (iv) d) None of these

91. If A and B are 3×3 matrices such that $AB = A$ and $BA = B$, then
a) $A^2 = A$ and $B^2 \neq B$ b) $A^2 \neq A$ and $B^2 = B$
c) $A^2 = A$ and $B^2 = B$ d) $A^2 \neq A$ and $B^2 \neq B$

92. Let A be a skew-symmetric matrix of even order, then $|A|$
a) Is a square b) Is not a square c) Is always zero d) None of these

93. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$, $B = (\text{adj } A)$ and $C = 5A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to
a) 5 b) 25 c) -1 d) 1

94. If $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ is the sum of a symmetric matrix B and skew-symmetric matrix, C , then B is
a) $\begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 5 & -2 \\ 2 & 2 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 6 & 6 & 7 \\ -6 & 2 & -5 \\ -7 & 5 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 6 & -2 \\ 2 & 0 & -2 \\ -2 & -2 & 0 \end{bmatrix}$

95. The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -2 & -2 & b \end{bmatrix}$ is a singular matrix, if b is equal to
a) -3 b) 3 c) 0 d) For any value of b

96. If A and B are square matrices of order 3×3 , then which of the following is true?
a) $AB = O \Rightarrow A = O$ or $B = O$ b) $\det(2AB) = 8 \det(A) \det(B)$

- c) $A^2 - B^2 = (A + B)(A - B)$ d) $\det(A + B) = \det(A) + \det(B)$
97. If $A = [a_{ij}]_{n \times n}$ be a diagonal matrix with diagonal element all different and $B = [b_{ij}]_{n \times n}$ be some another matrix. Let $AB = [c_{ij}]_{n \times n}$, then c_{ij} is equal to
- a) $a_{jj}b_{ij}$ b) $a_{ii}b_{ij}$ c) $a_{ij}b_{ij}$ d) $a_{ij}b_{ji}$
98. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 0 & 3 \\ 3 & -3 & 1 \end{bmatrix}$ be a matrix, then (determinant of A) $\times$ (adjoint of inverse A) is equal to
- a) $O_{3 \times 3}$ b) $\begin{bmatrix} 1 & 2 & -3 \\ -2 & 0 & 3 \\ 3 & -3 & 1 \end{bmatrix}$ c) I_3 d) $\begin{bmatrix} 3 & -3 & 1 \\ 3 & 0 & -2 \\ -1 & 2 & -3 \end{bmatrix}$
99. The rank of $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$ is
- a) 0 b) 1 c) 2 d) 3
100. Let a, b, c are positive real numbers. The following system of equations $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, -\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, in x, y and z has
- a) Infinite solutions b) Unique solution
c) No solution d) Finite number of solutions
101. If $A = [a_{ij}]_{m \times n}$ is a matrix of rank r and B is a square submatrix of order $r + 1$, then
- a) B is invertible
b) B is not invertible
c) B may or may not be invertible
d) None of these
102. If A is square matrix, A' , is its transpose, then $\frac{1}{2}(A - A')$ is
- a) A symmetric matrix b) A skew-symmetric matrix
c) A unit matrix d) An elementary matrix
103. Inverse of the matrix $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ is
- a) $\frac{1}{10} \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ b) $\frac{1}{10} \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$ d) $\frac{1}{10} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$
104. Let A be a matrix of rank r . Then,
- a) $\text{rank}(A^T) = r$ b) $\text{rank}(A^T) < r$ c) $\text{rank}(A^T) > r$ d) None of these
105. The adjoint matrix of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ is
- a) $\begin{bmatrix} 4 & 8 & 3 \\ 2 & 1 & 6 \\ 0 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ c) $\begin{bmatrix} 11 & 9 & 3 \\ 1 & 2 & 8 \\ 6 & 9 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 3 \\ -2 & 3 & -3 \end{bmatrix}$
106. If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then A^{-1} is equal to
- a) $-(3A^2 + 2A + 5)$ b) $3A^2 + 2A + 5$ c) $3A^2 - 2A - 5$ d) None of these
107. Let $A = [a_{ij}]_{n \times n}$ be a square matrix, and let c_{ij} be cofactor of a_{ij} in A . If $C = [c_{ij}]$, then
- a) $|C| = |A|$ b) $|C| = |A|^{n-1}$ c) $|C| = |A|^{n-2}$ d) None of these
108. The system of equations $x + y + z = 0, 2x + 3y + z = 0$ and $x = 2y = 0$ has
- a) A unique solution; $x = 0, y = 0, z = 0$ b) Infinite solutions
c) No solutions d) Finite number of non-zero solutions
109. If $2X - \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix}$, then X is equal to
- a) $\begin{bmatrix} 2 & 2 \\ 7 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ 7 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 2 \\ 7 & 1 \end{bmatrix}$ d) None of these
110. Let $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, then the values of x and y are

- a) $x = -\frac{1}{11}, y = \frac{2}{11}$ b) $x = -\frac{1}{11}, y = -\frac{2}{11}$ c) $x = \frac{1}{11}, y = \frac{2}{11}$ d) $x = \frac{1}{11}, y = -\frac{2}{11}$
111. Let A and B be two symmetric matrices of same order. Then, the matrix $AB - BA$ is
a) A symmetric matrix b) A skew-symmetric matrix
c) A null matrix d) The identity matrix
112. If $A = \begin{bmatrix} 1 & x \\ x^2 & 4y \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$ and $\text{adj } A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the values of x and y are respectively
a) (1, 1) b) (-1, 1) c) (1, 0) d) None of these
113. Let p is a non-singular matrix such that $1 + p + p^2 + \dots + p^n = O$ (O denotes the null matrix), then p^{-1} is
a) p^n b) $-p^n$ c) $-(1 + p + \dots + p^n)$ d) None of these
114. If $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$, then the value of $x + y + z$ is
a) 3 b) 0 c) 2 d) 1
115. The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is the matrix reflection in the line
a) $x = 1$ b) $x + y = 1$ c) $y = 1$ d) $x = y$
116. If $\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then
a) $a = 1, b = 1$ b) $a = \sin 2\theta, b = \cos 2\theta$
c) $a = \cos 2\theta, b = \sin 2\theta$ d) None of the above
117. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then $\text{adj } A$ is equal to
a) A b) A' c) $3A$ d) $3A'$
118. Let the homogeneous system of linear equations $px + y + z = 0$, $x + qy + z = 0$, and $x + y + rz = 0$, where $p, q, r \neq 1$, have a non-zero solution, then the value of $\frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r}$ is
a) -1 b) 0 c) 2 d) 1
119. If $A = \begin{bmatrix} 1 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then B is equal to
a) $\cos^2 \frac{\theta}{2} \cdot A$ b) $\cos^2 \frac{\theta}{2} \cdot A^T$ c) $\cos^2 \theta \cdot I$ d) $\sin^2 \frac{\theta}{2} \cdot A$
120. The values of x, y, z in order, if the system of equations $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$ has unique solution, are
a) 2, 1, 5 b) 1, 1, 1 c) 1, -2, -1 d) 1, 2, -1
121. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix, then for $n \geq 2$, A^n is equal to
a) $nA - (n-1)I$ b) $nA - I$ c) $2^{n-1}A - (n-1)I$ d) $2^{n-1}A - I$
122. Matrix M_r is defined as $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, $r \in N$ value of $\det(M_1) + \det(M_2) + \det(M_3) + \dots + \det(M_{2007})$ is
a) 2007 b) 2008 c) 2008^2 d) 2007^2
123. The number of solutions of the system of equations $x_2 - x_3 = 1$, $-x_1 + 2x_3 = -2$, $x_1 - 2, x_1 - 2x_2 = 3$ is
a) Zero b) One c) Two d) Infinite
124. If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ii} = k$ for all i , then trace of A is equal to
a) nk b) $n + k$ c) n/k d) None of these
125. If $D = \text{diag}[d_1, d_2, d_3, \dots, d_n]$, where $d_i \neq 0 \forall i = 1, 2, \dots, n$ then D^{-1} is equal to
a) O b) I_n
c) $\text{diag}[d_1^{-1}, d_2^{-1}, \dots, d_n^{-1}]$ d) None of the above
126. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then $\lim_{n \rightarrow \infty} \frac{1}{n} A^n$ is
a) $\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ d) None of these

127. The system of equations $2x + y - 5 = 0, x - 2y + 1 = 0, 2x - 14y - a = 0$, is consistent. Then, a is equal to
- a) 1 b) 2 c) 5 d) None of these
128. The system of equation
 $ax + y + z = \alpha - 1$
 $x + \alpha y + z = \alpha - 1$
 $x + y + \alpha z = \alpha - 1$
 Has no solution, if α is
- a) 1 b) Not-2 c) Either-2 or 1 d) -2
129. A matrix $A = [a_{ij}]$ is an upper triangular matrix, if
- a) It is a square matrix and $a_{ij} = 0, i < j$
 b) It is a square matrix and $a_{ij} = 0, i > j$
 c) It is not a square matrix and $a_{ij} = 0, i > j$
 d) It is not a square matrix and $a_{ij} = 0, i < j$
130. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x is equal to
- a) -1 b) 0 c) 1 d) 2
131. $A = \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}$ and $A^{-1} = \lambda (\text{adj } A)$, then λ equal to
- a) $-\frac{1}{6}$ b) $\frac{1}{3}$ c) $-\frac{1}{3}$ d) $\frac{1}{6}$
132. If $A = [a_{ij}]$ is a 4×4 matrix and C_{ij} is the cofactor of the element a_{ij} in $|A|$, then the expression $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14}$ is equal to
- a) 0 b) -1 c) 1 d) $|A|$
133. For what value of λ , the system of equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = 10$ is consistent?
- a) 1 b) 2 c) -1 d) 3
134. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then A^{100} is equal to
- a) $2^{100}A$ b) $2^{99}A$ c) $100A$ d) $299A$
135. Inverse of the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ is
- a) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ b) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$ c) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ d) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$
136. Which of the following is correct?
- a) Determinant is square matrix
 b) Determinant is a number associated to a matrix
 c) Determinant is a number associated to a square matrix
 d) None of these
137. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then B equals
- a) $I \cos \theta + J \sin \theta$ b) $I \sin \theta + J \cos \theta$ c) $I \cos \theta - J \sin \theta$ d) $-I \cos \theta + J \sin \theta$
138. What must be the matrix X if $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$?
- a) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$
139. A and B be 3×3 matrices. Then, $AB = O$ implies
- a) $A = O$ and $B = O$
 b) $|A| = 0$ and $|B| = 0$
 c) Either $|A| = 0$ or $|B| = 0$
 d) $A = O$ or $B = O$
140. Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, D = \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 1 \\ 4 & -1 & -2 \end{bmatrix}$, if $X = A^{-1}D$, then X is equal to

a) $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

b) $\begin{bmatrix} \frac{8}{3} \\ -1 \\ \frac{3}{0} \end{bmatrix}$

c) $\begin{bmatrix} \frac{8}{3} \\ -1 \\ 1 \\ 0 \end{bmatrix}$

d) $\begin{bmatrix} \frac{8}{3} \\ 1 \\ \frac{3}{-1} \end{bmatrix}$

141. If A and B are matrices such that AB and $A + B$ both are defined, then

- a) A and B can be any two matrices
- b) A and B are square matrices not necessarily of the same order
- c) A, B are square matrices of the same order
- d) Number of columns of A is same as the number of rows of B

142. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz, y = az + cx$, and $z = bx + ay$ have non-zero solution. Then, $a^2 + b^2 + c^2 + 2abc$ is equal to

- a) 1
- b) 2
- c) -1
- d) 0

143. If I_n is the identity matrix of order n , then rank of I_n is

- a) 1
- b) n
- c) 0
- d) None of these

144. If the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{bmatrix}$ is singular, then λ is equal to

- a) 3
- b) 4
- c) 2
- d) 5

145. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $I + A + A^2 + A^3 + \dots \infty$ equals to

- a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- b) $\begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$
- c) $\begin{bmatrix} 1/2 & -1/3 \\ -1/2 & 0 \end{bmatrix}$
- d) $\begin{bmatrix} -1/4 & 1/3 \\ 1/2 & 0 \end{bmatrix}$

146. If A is a non-singular square matrix of order n , then the rank of A is

- a) Equal to n
- b) Less than n
- c) Greater than n
- d) None of these

147. If $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$ and $f(t) = t^2 - 3t + 7$, then $f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix}$ is equal to

- a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- d) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

148. The system of linear equations

$$x + y + z = 2$$

$$2x + y - z = 3$$

$$3x + 2y + kz = 4$$
 has a unique solution if

- a) $k \neq 0$
- b) $-1 < k < 1$
- c) $-2 < k < 2$
- d) $k = 0$

149. The number of solutions of the system of equations

$$2x + y - z = 7, x - 3y + 2z = 1, x + 4y - 3z = 5$$
 is

- a) 0
- b) 1
- c) 2
- d) 3

150. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, the value of X^n is equal to

- a) $\begin{bmatrix} 3^n & -4^n \\ n & -n \end{bmatrix}$
- b) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$
- c) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$
- d) None of these

151. If I_3 is the identity matrix of order 3, then $(I_3)^{-1} =$

- a) 0
- b) $3I_3$
- c) I_3
- d) Not necessarily exists

152. If $A = [a_{ij}]$ is a square matrix of order $n \times n$ and k is a scalar, then $|kA| =$

- a) $k^n |A|$
- b) $k|A|$
- c) $k^{n-1} |A|$
- d) None of these

153. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to

- a) Null matrix
- b) Unit matrix
- c) $-A$
- d) A

154. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$ is

- a) 1
- b) -1
- c) 4
- d) No real values

155. If A is a square matrix such that $A(\text{adj } A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then $|\text{adj } A| =$

- a) 4 b) 16 c) 64 d) 256

156. If ω is a complex cube root of unity and $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then A^{50} is
a) $\omega^2 A$ b) ωA c) A d) 0

157. If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$, then $x + y$ equals
a) 0 b) -1 c) 2 d) None of these

158. The adjoint of the matrix $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ is
a) $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ b) $\begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$ c) $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ d) $\begin{bmatrix} -\sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

159. The inverse matrix of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ is
a) $\begin{bmatrix} \frac{1}{2} & -\frac{11}{22} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{31}{22} \\ \frac{1}{2} & -\frac{2}{22} \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{2} & -4 & \frac{5}{2} \\ 1 & -6 & 3 \\ 1 & 2 & -1 \end{bmatrix}$ c) $\frac{1}{2} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ d) $\frac{1}{2} \begin{bmatrix} 1 & -1 & -1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$

160. If $f(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $\{f(\theta)^{-1}\}$ is equal to
a) $f(-\theta)$ b) $f(\theta)^{-1}$ c) $f(2\theta)$ d) None of these

161. If the three linear equations
 $x + 4ay + az = 0$
 $x + 3by + bz = 0$
 $x + 2cy + cz = 0$
Have a non-trivial solution, where $a \neq 0, b \neq 0, c \neq 0$, then $ab + bc$ is equal to
a) $2ac$ b) $-ac$ c) ac d) $-2ac$

162. If A and B are two matrices such that rank of $A = m$ and rank of $B = n$, then
a) rank $(AB) = mn$
b) rank $(AB) \geq \text{rank}(A)$
c) rank $(AB) \geq \text{rank}(B)$
d) rank $(AB) \leq \min(\text{rank } A, \text{rank } B)$

163. If A is a non-zero column matrix of order $m \times 1$ and B is a non-zero row matrix of order $1 \times n$, then rank of AB equates
a) 1 b) 2 c) 3 d) 4

164. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A is equal to
a) $-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

165. If $A^2 - A + I = 0$, then the inverse of A is
a) $I - A$ b) $A - I$ c) A d) $A + I$

166. If B is an invertible matrix and A is a matrix, then
a) rank $(BA) = \text{rank}(A)$ b) rank $(BA) \geq \text{rank}(B)$ c) rank $(BA) > \text{rank}(A)$ d) rank $(BA) > \text{rank}(B)$

167. If $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$, $|\text{adj } A|$ is equal to
a) 6 b) 16 c) 10 d) None of these

168. $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$ is equal to
a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

169. Let $A = [a_{ij}]_{m \times n}$ be a matrix such that $a_{ij} = 1$ for all i, j . Then,
a) rank $(A^T) > 1$ b) rank $(A) = 1$ c) rank $(A) = m$ d) rank $(A) = n$

170. Let A be a square matrix all of whose entries are integers. Then, which one of the following is true?

- a) If $\det(A) = \pm 1$, then A^{-1} need not exist
- b) If $\det(A) = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
- c) If $\det(A) \neq \pm 1$, then A^{-1} exists and all its entries are non – integers
- d) If $\det(A) = \pm 1$, then A^{-1} exists and all its entries are integers

171. Matrix $A = \begin{bmatrix} 1 & 0 & -k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ is invertible for

- a) $k = 1$
- b) $k = -1$
- c) $k = \pm 1$
- d) None of these

172. If $\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

- a) $a = 1, b = 1$
- b) $a = \cos 2\theta, b = \sin 2\theta$
- c) $a = \sin 2\theta, b = \cos 2\theta$
- d) None of these

173. If $x^2 + y^2 + z^2 \neq 0, x = cy + bz, y = az + cx$ and $z = bx + ay$, then $a^2 + b^2 + c^2 + 2abc =$

- a) 2
- b) $a + b + c$
- c) 1
- d) $ab + bc + ca$

174. If $A = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$ and $A^2 - 4A = 10I = A$ then k is equal to

- a) 0
- b) -4
- c) 4 and not 1
- d) 1 or 4

175. Matrix A such that $A^2 = 2A - I$, where I is the identity matrix. Then, for $n \geq 2, A^n$ is equal to

- a) $nA - (n - 1)I$
- b) $nA - I$
- c) $2^{n-1}A - (n - 1)I$
- d) $2^{n-1}A - I$

176. The matrix A satisfying the equation $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is

- a) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$
- b) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$
- c) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$
- d) None of these

177. If A is an orthogonal matrix, then A^{-1} equals

- a) A
- b) A^T
- c) A^2
- d) None of these

178. By elementary transformation method, the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$ is

- a) $\begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$
- b) $\begin{bmatrix} 2 & 0 & -1 \\ 0 & -3 & 2 \\ -1 & 2 & -1 \end{bmatrix}$
- c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$
- d) None of these

179. What must be the matrix X , if $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$?

- a) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$
- b) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$
- c) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$
- d) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$

180. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then $x =$

- a) 3
- b) 5
- c) 2
- d) 4

181. If $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$, A^4 is equal to

- a) $27A$
- b) $81A$
- c) $243A$
- d) $729A$

182. If ω is a complex cube root of unity, then the matrix $A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}$ is a

- a) Singular matrix
- b) Non-symmetric matrix
- c) Skew-symmetric matrix
- d) None of these

183. The values of λ and μ for which of the system of equations $x + y + z = 6, x = 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have infinite number of solutions, are

- a) $\lambda = 3, \mu = 10$
- b) $\lambda = 3, \mu \neq 10$
- c) $\lambda \neq 3, \mu = 10$
- d) $\lambda \neq 3, \mu \neq 10$

184. If A and B are square matrices of the same order such that $(A + B)(A - B) = A^2 -$

- B^2 , then $(ABA^{-1})^2$ is equal to
a) B^2 b) I c) A^2B^2 d) A^2
185. If A is a skew-symmetric matrix, then trace of A is
a) 1 b) -1 c) 0 d) None of these
186. A square matrix P satisfies $P^2 = I - P$, where I is the identity matrix. If $P^n = 5I - 8P$, then n is equal to
a) 4 b) 5 c) 6 d) 7
187. Let A and B are two square matrices such that $AB = A$ and $BA = B$, then A^2 equals to
a) B b) A c) I d) O
188. A and B are two square matrices of same order and A' denotes the transpose of A , then
a) $(AB) = B'A'$
b) $(AB)' = A'B'$
c) $AB = 0 \Rightarrow |A| = 0$ or $|B| = 0$
d) $AB = 0 \Rightarrow A = 0$ or $B = 0$
189. The element in the first row and third column of the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is
a) -2 b) 0 c) 1 d) 7
190. If $A = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x)$, then A^{-1} is equal to
a) $f(-x)$ b) $f(x)$ c) $-f(x)$ d) $-f(-x)$
191. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then A^{-1} is
a) $-A$ b) A c) 1 d) None of these
192. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I_2 = O$, then the value of k is
a) 3 b) 5 c) 7 d) -7
193. Consider the following statements :
1. There can exist two matrices A, B of order 2×2 such that $AB - BA = I_2$
2. Positive odd integral power of a skew-symmetric matrix is symmetric
a) Only (1) b) Only (2) c) Both of these d) None of these
194. If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$, then $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is equal to
a) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ c) $\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$
195. The number of non-trivial solutions of the system $x - y + z = 0$, $x + 2y - z = 0$, $2x + y + 3z = 0$ is
a) 0 b) 1 c) 2 d) 3
196. If $\begin{bmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{bmatrix}$ has no inverse, then the real value of x is
a) 2 b) 3 c) 0 d) 1
197. If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then x is
a) $\frac{13}{25}$ b) $-\frac{25}{13}$ c) $\frac{5}{13}$ d) $\frac{25}{13}$
198. The rank of the matrix $A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$ is
a) 2 b) 3 c) 1 d) Indeterminate
199. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

- a) $\alpha = a^2 + b^2, \beta = ab$
 b) $\alpha = a^2 + b^2, \beta = 2ab$
 c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$
 d) $\alpha = 2ab, \beta = a^2 + b^2$

200. If $A = [a_{ij}]$ is a scalar matrix, then trace of A is

- a) $\sum_i \sum_j a_{ij}$ b) $\sum_i a_{ij}$ c) $\sum_j a_{ij}$ d) $\sum_i a_{ii}$

201. The system of equations $x + 2y + 3z = 1, 2x + y + 3z = 2, 5x + 5y + 9z = 5$ has

- a) Unique solution b) Infinite many solution
 c) Inconsistent d) None of the above

202. The rank of the matrix $\begin{bmatrix} 4 & 2 & (1-x) \\ 5 & k & 1 \\ 6 & 3 & (1+x) \end{bmatrix}$ is 2, then

- a) $k = \frac{5}{2}, x = \frac{1}{5}$ b) $k = \frac{5}{2}, x \neq \frac{1}{5}$ c) $k = \frac{1}{5}, x = \frac{5}{2}$ d) None of these

203. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A^2 =$

- a) $\begin{bmatrix} 8 & -5 \\ -5 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 8 & -5 \\ 5 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 8 & -5 \\ -5 & -3 \end{bmatrix}$ d) $\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

204. If ω is a root of unity and $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$, then A^{-1} is equal to

- a) $\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}$ b) $\frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ c) $\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$ d) $\frac{1}{2} \begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$

205. If $A = [a_{ij}]_{m \times n}$ is a matrix of rank r , then

- a) $r = \min(m, n)$ b) $r < \min(m, n)$ c) $r \leq \min(m, n)$ d) None of these

206. For each real $x: -1 < x < 1$. Let $A(x)$ be the matrix $(1-x)^{-1} \begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \frac{x+y}{1+xy}$, then

- a) $A(z) = A(x)A(y)$ b) $A(z) = A(x) - A(y)$ c) $A(z) = A(x)[A(y)]^{-1}$ d) $A(z) = A(x) + A(y)$

207. If $A(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then the matrix $A^2(\alpha)$ is

- a) $A(2\alpha)$ b) $A(\alpha)$ c) $A(3\alpha)$ d) $A(4\alpha)$

208. If A is a symmetric matrix and $n \in \mathbb{N}$, then A^n is

- a) Symmetric matrix b) A diagonal matrix
 c) Skew-symmetric matrix d) None of the above

209. The inverse matrix of $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ is

- a) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{2} & -4 & \frac{5}{2} \\ \frac{1}{2} & -6 & 3 \\ 1 & 2 & -1 \end{bmatrix}$ c) $\frac{1}{2} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ d) $\frac{1}{2} \begin{bmatrix} 1 & -1 & -1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$

210. If $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ is expressed as the sum of a symmetric and skew-symmetric matrix, then the symmetric matrix is

- a) $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

211. If the system of linear equations $x + 2ay + az = 0, x + 3by + bz = 0$ and $x + 4cy + cz = 0$ has a non-zero solution, then a, b, c

- a) Are in AP b) Are in GP
 c) Are in HP d) Satisfy $a + 2b + 3c = 0$

212. For what value of k the following system of linear equations will have infinite solutions
 $x - y + z = 3, 2x + y - z = 2$
and $-3x + 2ky + 6z = 3$
a) $k \neq 2$ b) $k = 0$ c) $k = 3$ d) $k \in [2, 3]$
213. The product of two orthogonal matrices is
a) Orthogonal b) Involutory c) Unitary d) Idempotent
214. The system of equations $x + y + z = 8, x - y + 2z = 6, 3x + 5y - 7z = 14$ has
a) No solution b) Unique solution
c) Infinitely many solution d) None of the above
215. If the system of equations $x + ay = 0, az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is
a) -1 b) 1 c) 0 d) No real values
216. $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}^{-1} =$
a) $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}$ c) $\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ d) $\begin{bmatrix} 6 & -5 \\ 7 & -6 \end{bmatrix}$
217. Let $F(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $[F(\alpha)]^{-1}$ is equal to
a) $F(-\alpha)$ b) $F(\alpha^{-1})$ c) $F(2\alpha)$ d) None of these
218. Let for any matrix M, M^{-1} exist which of the following is not true?
a) $|M^{-1}| = |M|^{-1}$ b) $(M^2)^{-1} = (M^{-1})^2$ c) $(M^T)^{-1} = (M^{-1})^T$ d) $(M^{-1})^{-1} = M$
219. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?
a) $AB = BA$ b) Either of A or B is a zero matrix
c) Either of A or B is an identity matrix d) $A = B$
220. $x_1 + 2x_2 + 3x_3 = 2x_1 + 3x_2 + x_3 = 3x_1 + x_2 + 2x_3 = 0$.
This system of equations has
a) Infinite solution b) No solution c) No solution d) Unique solution
221. If A is a 3×4 matrix and B is a matrix such that $A^T B$ and BA^T are both defined, then order of B is
a) 3×4 b) 3×3 c) 4×4 d) 4×3
222. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then the value of X^n is
a) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ b) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$ c) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ d) None of these
223. Let $f(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\alpha \in R$. Then, $(F(\alpha))^{-1}$ is equal to
a) $F(-\alpha)$ b) $F(\alpha^{-1})$ c) $F(2\alpha)$ d) None of these
224. For any square matrix A, AA^T is a
a) Unit matrix b) Symmetric matrix
c) Skew-symmetric matrix d) Diagonal matrix
225. If A is a square matrix of order $n \times n$, then $\text{adj}(\text{adj } A)$ is equal to
a) $|A|^n A$ b) $|A|^{n-1} A$ c) $|A|^{n-2} A$ d) $|A|^{n-3} A$
226. If a system of the equations $(\alpha + 1)^3 x + (\alpha + 2)^3 y - (\alpha + 3)^3 = 0$,
 $(\alpha + 1)x + (\alpha + 2)y - (\alpha + 3) = 0$, and $x + y - 1 = 0$ is consistent. What is the value of α ?
a) 1 b) 0 c) -3 d) -2
227. If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then $\lim_{n \rightarrow \infty} \frac{1}{n} A^n$ is
a) A null matrix b) An identity matrix c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ d) None of these
228. If $A = \begin{bmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha \end{bmatrix}$

and, $B = \begin{bmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{bmatrix}$

are two matrices such that the product AB is the null matrix, then $(\alpha - \beta)$ is

- a) 0
- b) Multiple of π
- c) An odd multiple of $\pi/2$
- d) None of these

229. If A be a square matrix of order n and if $|A| = D$ and $|\text{adj } A| = D'$, then

- a) $DD' = D^2$
- b) $DD' = D^{-1}$
- c) $DD' = D^n$
- d) None of these

230. If $1, \omega, \omega^2$ are the cube roots of unity and if

$$\begin{bmatrix} 1 + \omega & 2\omega \\ -2\omega & -b \end{bmatrix} + \begin{bmatrix} a - \omega & \\ 3\omega & 2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix}, \text{ then } a^2 + b^2 \text{ is equal to}$$

- a) $1 + \omega^2$
- b) $\omega^2 - 1$
- c) $1 + \omega$
- d) $(1 + \omega)^2$

231. If a square matrix A is orthogonal as well as symmetric, then

- a) A is involutory matrix
- b) A is idempotent matrix
- c) A is a diagonal matrix
- d) None of these

232. The real value of k for which the system of equations $2kx - 2y + 3z = 0, x + ky + 2z = 0, 2x + kz = 0$, has non-trivial solution is

- a) 2
- b) -2
- c) 3
- d) -3

233. If the matrices $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$, then AB

- a) $\begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}$
- b) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$
- c) $\begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$
- d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

234. If $A = \begin{bmatrix} a & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$ is

- a) 1
- b) -1
- c) 4
- d) No real values

235. If $E(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then $E(\alpha)E(\beta)$ is equal to

- a) $E(0^\circ)$
- b) $E(\alpha\beta)$
- c) $E(\alpha + \beta)$
- d) $E(\alpha - \beta)$

236. If $A = \begin{bmatrix} b & b^2 \\ -a^2 & -ab \end{bmatrix}$, then A is

- a) Idempotent
- b) Involutory
- c) Nilpotent
- d) Scalar

237. The matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ is

- a) Unitary
- b) Orthogonal
- c) Nilpotent
- d) Involutory

238. Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. If $AX = B$, then X is equal to

- a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- b) $\begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$
- c) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$
- d) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

239. If A is a skew-symmetric matrix of odd order, then $|\text{adj } A|$ is equal to

- a) 0
- b) n
- c) n^2
- d) None of these

240. The system of equations $x + 3y + 2z = 0, 3x + y + z = 0$ and $2x - 2y - z = 0$

- a) Possesses a trivial solution only
- b) Possesses a non-zero unique solution
- c) Does not have a common non-zero solution
- d) Has infinitely many solutions

241. Consider the following statements:

1. A square matrix A is hermitian, if $A = A'$
2. Let $A = [a_{ij}]$ be a skew-hermitian matrix, then a_{ij} is purely imaginary
3. All integer powers of a symmetric matrix are symmetric. Which of these is/are correct?

- a) (1) and (2) b) (2) and (3) c) (3) and (1) d) (1), (2) and (3)
242. If $a_1, a_2, a_3, a_4, a_5, a_6$ are in AP with common difference $d \neq 0$, then the system of equations $a_1x + a_2y = a_3, a_4x + a_5y = a_6$ has
- a) Infinite number of solutions b) Unique solution
c) No solution d) Cannot say any thing
243. If I_n is the identity matrix of order n , then $(I_n)^{-1}$ is equal to
- a) Does not exist b) I_n c) 0 d) nI_n
244. If A is a square matrix, then $\text{adj } A^T - (\text{adj } A)^T$ is equal to
- a) $2|A|$ b) $2|A|I$ c) Null matrix d) Unit matrix
245. If $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 10 \end{bmatrix}$, then $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is equal to
- a) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ d) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$
246. Consider the system of equations
- $$a_1x + b_1y + c_1z = 0$$
- $$a_2x + b_2y + c_2z = 0$$
- $$a_3x + b_3y + c_3z = 0$$
- if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the system has
- a) More than two solutions
b) One trivial and one non-trivial solutions
c) No solution
d) Only trivial solution (0,0,0)
247. The number of solutions of the system of equations
- $$x - y + z = 2$$
- $$2x + y - z = 5$$
- $$4x + y + z = 10$$
- is
- a) ∞ b) 1 c) 2 d) 0
248. If $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then which of the following statement is not correct?
- a) A is orthogonal matrix b) A' is orthogonal matrix
c) Determinant $A = 1$ d) A is not invertible
249. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is equal to
- a) $2A$ b) A c) $-A$ d) I
250. The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is
- a) 1 if $a = 6$ b) 2 if $a = 1$ c) 3 if $a = 2$ d) 4 if $a = -6$
251. If $D = \text{diag}(d_1, d_2, d_3, \dots, d_n)$, where $d_i \neq 0$ for all $i = 1, 2, \dots, n$, then D^{-1} is equal to
- a) D b) $\text{diag}(d_1^{-1}d_2^{-1}, \dots, d_n^{-1})$ c) I_n d) None of these
252. If $f(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, then $f(A)$ is equal to
- a) $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$
253. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the unit matrix of order 2, then A^2 equals
- a) $4A - 3I$ b) $3A - 4I$ c) $A - I$ d) $A + I$
254. Which one of the following is true always for any two non-singular matrices A and B of same order?
- a) $AB = BA$ b) $(AB)^t = A^tB^t$

c) $(A + B)(A - B) = A^2 - B^2$

d) $(AB)^{-1} = B^{-1}A^{-1}$

255. The inverse of $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 1/2 & 0 & 2 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 1/2 \\ 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \end{bmatrix}$

256. The values of a for which the system of equations $ax + y + z = 0$, $x - ay + z = 0$, $x + y + z = 0$ possesses non-zero solution, are given by

a) 1, 2

b) 1, -1

c) 0

d) None of these

257. If A is square matrix, then

a) $A + A^T$ is symmetric

b) AA^T is skew-symmetric

c) $A^T + A$ is skew-symmetric

d) $A^T A$ is skew-symmetric

258. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then $\det [\text{adj}(\text{adj } A)]$ is equal to

a) 12^4

b) 13^4

c) 14^4

d) None of these

259. If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$, then $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is equal to

a) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

c) $\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

260. If $A = \begin{bmatrix} 1 & 2 \\ -4 & -1 \end{bmatrix}$, then A^{-1} is

a) $\frac{1}{7} \begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$

b) $\frac{1}{7} \begin{bmatrix} 1 & 2 \\ -4 & -1 \end{bmatrix}$

c) $\frac{1}{7} \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$

d) Does not exist

261. If the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is commutative with the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then

a) $a = 0, b = c$

b) $b = 0, c = d$

c) $c = 0, d = a$

d) $d = 0, a = b$

262. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, then AB is equal to

a) $\begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 11 & 4 & 3 \\ 1 & 2 & 3 \\ 0 & 3 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 8 & 4 \\ 2 & 9 & 6 \\ 0 & 2 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 & 2 \\ 5 & 4 & 3 \\ 1 & 8 & 2 \end{bmatrix}$

263. Let A be a skew-symmetric matrix of odd order, then $|A|$ is equal to

a) 0

b) 1

c) -1

d) None of these

264. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2005} P$ is

a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 2005 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

265. If X and Y are 2×2 matrices such that $2X + 3Y = O$ and $X + 2Y = I$, where O and I denote the 2×2 zero matrix and the 2×2 identity matrix, then X is equal to

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

c) $\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

266. Consider the system of linear equations

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

- a) Infinite number of solutions
b) Exactly 3 solutions
c) A unique solution
d) No solution
267. If $A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$, then the value of x is
a) 2
b) 3
c) -4
d) 4
268. If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is
a) A is a zero matrix
b) $A = (-1)I$, where I is a unit matrix
c) A^{-1} does not exist
d) $A^2 = I$
269. The inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$ is equal to
a) $\begin{bmatrix} 10 & 3 \\ 3 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$
c) $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$
d) $\begin{bmatrix} -1 & -3 \\ -3 & -10 \end{bmatrix}$
270. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, then AB is equal to
a) $\begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$
b) $\begin{bmatrix} 11 & 4 & 3 \\ 1 & 2 & 3 \\ 0 & 3 & 3 \end{bmatrix}$
c) $\begin{bmatrix} 1 & 8 & 4 \\ 2 & 9 & 6 \\ 0 & 2 & 0 \end{bmatrix}$
d) $\begin{bmatrix} 0 & 1 & 2 \\ 5 & 4 & 3 \\ 1 & 8 & 2 \end{bmatrix}$
271. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(y) = \begin{bmatrix} \cos y & 0 & \sin y \\ 0 & 1 & 0 \\ -\sin y & 0 & \cos y \end{bmatrix}$, then $[F(x)G(y)]^{-1}$ is equal to
a) $F(-x)G(-y)$
b) $F(x^{-1})G(y^{-1})$
c) $G(-y)F(-x)$
d) $G(y^{-1})F(x^{-1})$
272. Let $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, then the value of x and y are
a) $x = \frac{-1}{11}, y = \frac{2}{11}$
b) $x = \frac{-1}{11}, y = \frac{-2}{11}$
c) $x = \frac{1}{11}, y = \frac{2}{11}$
d) $x = \frac{1}{11}, y = \frac{-2}{11}$
273. If A^T, B^T are transpose matrices of the square matrices A, B respectively, then $(AB)^T$ is equal to
a) $A^T B^T$
b) AB^T
c) BA^T
d) $B^T A^T$
274. If $\begin{bmatrix} x+y+z \\ x+y \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$, then the value of (x, y, z) is
a) $(4, 3, 2)$
b) $(3, 2, 4)$
c) $(2, 3, 4)$
d) None of the above
275. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, then A is equal to
a) Idempotent
b) Involutory
c) Nilpotent
d) Scalar
276. For non-singular square matrices A, B and C of the same order, $(AB^{-1}C)^{-1}$ is equal to
a) $A^{-1}BC^{-2}$
b) $C^{-1}B^{-1}A^{-1}$
c) CBA^{-1}
d) $C^{-1}BA^{-1}$
277. The matrix $\begin{bmatrix} \lambda - 1 & 4 \\ -3 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible, if
a) $\lambda \neq -17$
b) $\lambda \neq -18$
c) $\lambda \neq -19$
d) $\lambda \neq -20$
278. If a, b, c are non-zero, then the number of solutions of following system of equation is
 $\frac{2x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \dots(i)$
 $-\frac{x^2}{a^2} + \frac{2y^2}{b^2} - \frac{z^2}{c^2} = 0 \dots(ii)$
 $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{2z^2}{c^2} = 0 \dots(iii)$
a) 6
b) 8
c) 9
d) Infinite
279. If $A = \begin{bmatrix} 1 & \log_b a \\ \log_a b & 1 \end{bmatrix}$, then $|A|$ is equal to
a) 1
b) 0
c) $\log_a b$
d) $\log_b a$
280. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then A^{-1} is equal to

281. a) $-\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ b) $\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ c) $\begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$
 If $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$, then PQ is equal to
282. a) $\begin{bmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = (\text{adj } A)$, and $C = 5A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to
283. For $0 < \theta < \pi$, if $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then
 a) $A^T = A$ b) $A^T = -A$ c) $A^2 = I$ d) $A^T = A^{-1}$
284. The values of a for which the system of equations $x + y + z = 0, x + ay + az = 0, x - ay + z = 0$, possesses non-zero solutions, are given by
 a) 1, 2 b) 1, -1 c) 1, 0 d) None of these
285. If $x \begin{bmatrix} -3 \\ 4 \end{bmatrix} + y \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \end{bmatrix}$, then
 a) $x = -2, y = 1$ b) $x = -9, y = 10$ c) $x = 22, y = 1$ d) $x = 2, y = -1$
286. If A is a square matrix such that $AA^T = I = A^T A$, then A is
 a) A symmetric matrix
 b) A skew-symmetric matrix
 c) A diagonal matrix
 d) An orthogonal matrix
287. The inverse of the matrix $\begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$ is
 a) $\frac{1}{11}\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$ c) $\frac{1}{13}\begin{bmatrix} -2 & 5 \\ 1 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$
288. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^5 =$
 a) $5A$ b) $10A$ c) $16A$ d) $32A$
289. If $A(\theta) = \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}$ and $AB = I$, then $(\sec^2 \theta)B$ is equal to
 a) $A(\theta)$ b) $A\left(\frac{\theta}{2}\right)$ c) $A(-\theta)$ d) $A\left(-\frac{\theta}{2}\right)$
290. If $A = [a_{ij}]$ is a skew-symmetric matrix of order n , then $a_{ii} =$
 a) 0 for some i b) 0 for all $i = 1, 2, \dots, n$ c) 1 for some i d) 1 for all $i = 1, 2, \dots, n$
291. Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then A^n is equal to
 a) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$ b) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$ d) $\begin{bmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & na \end{bmatrix}$
292. If A, B are symmetric matrices of the same order then $AB - BA$ is
 a) Symmetric matrix
 b) Skew-symmetric matrix
 c) Null matrix
 d) Unit matrix
293. If A is any $m \times n$ matrix such that AB and BA are both defined, then B is an
 a) $m \times n$ matrix b) $n \times m$ matrix c) $n \times n$ matrix d) $m \times m$ matrix
294. If A is a square matrix of order $n \times n$ and k is a scalar, then $\text{adj } (kA)$ is equal to
 a) $k \text{ adj } A$ b) $k^n \text{ adj } A$ c) $k^{n-1} \text{ adj } A$ d) $k^{n+1} \text{ adj } A$
295. $x + ky - z = 0, 3x - ky - z = 0$ and $x - 3y + z = 0$ has non-zero solution for k is equal to

- a) -1 b) 0 c) 1 d) 2
296. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, I is the unit matrix of order 2 and a, b are arbitrary constants, then $(aI + bA)^2$ is equal to
a) $a^2I - abA$ b) $a^2I + 2abA$ c) $a^2I + b^2A$ d) None of the above
297. If A is an orthogonal matrix, then
a) $|A| = 0$ b) $|A| = \pm 1$ c) $|A| = \pm 2$ d) None of these
298. Given $2x - y + 2z = 2, x - 2y + 2z = -4, x + y + \lambda z = 4$ then the value of λ such that the given system of equations has no solution, is
a) 3 b) 1 c) 0 d) -3
299. If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$ is an idempotent matrix, then x is equal to
a) -5 b) -1 c) -3 d) -4
300. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and $\text{adj } A = \begin{bmatrix} 6 & -2 & -6 \\ -4 & 2 & x \\ y & -1 & -1 \end{bmatrix}$, then $x + y =$
a) 6 b) -1 c) 3 d) 1
301. If $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$, then the value of x, y, z, w are
a) 2, 2, 3, 4 b) 2, 3, 1, 2 c) 3, 3, 0, 1 d) None of these
302. If for a matrix $A, A^2 + I = O$, where I is the identity matrix, then A equals
a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
303. If $m[-3 \ 4] + n[4 \ -3] = [10 \ -11]$, then $3m + 7n$ is equal to
a) 3 b) 5 c) 10 d) 1
304. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and $A(\text{adj } A) = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of k is
a) $\sin x \cos x$ b) 1 c) 2 d) 3
305. If $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, then $(A^{-1})^3$ is equal to
a) $\frac{1}{27} \begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$ b) $\frac{1}{27} \begin{bmatrix} -1 & 26 \\ 0 & 27 \end{bmatrix}$ c) $\frac{1}{27} \begin{bmatrix} 1 & -26 \\ 0 & -27 \end{bmatrix}$ d) $\frac{1}{27} \begin{bmatrix} -1 & -26 \\ 0 & -27 \end{bmatrix}$
306. Let $M = [a_{uv}]_{n \times n}$ be a matrix, where $a_{uv} = \sin(\theta_u - \theta_v) + i \cos(\theta_u - \theta_v)$, the M is equal to
a) $\bar{M}$ b) $-M$ c) $\bar{M}^T$ d) $-\bar{M}^T$
307. If the system of homogeneous equations $2x - y + z = 0, x - 2y + z = 0, \lambda x - y + 2z = 0$ has infinitely many solutions, then
a) $\lambda = 5$ b) $\lambda = -5$ c) $\lambda \neq \pm 5$ d) None of these
308. Assuming that the sum and product given below are defined, which of the following is not true for matrices?
a) $A + B = B + A$ b) $AB = AC$ does not imply $B = C$
c) $AB = O$ implies $A = O$ or $B = O$ d) $(AB)' = B'A'$
309. If $E(\theta) = \begin{bmatrix} \cos^2 \theta & \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta & \theta \end{bmatrix}$ and θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$, then $E(\theta) E(\phi)$ is a
a) Unit matrix b) Null matrix c) Diagonal matrix d) None of these
310. Consider the system of equations in x, y, z as
 $x \sin 3\theta - y + z = 0$
 $x \cos 2\theta + 4y + 3z = 0$
and $2x + 7y + 7z = 0$
If this system has a non-trivial solution, then for integer n , values of θ are given by
a) $\pi \left(n + \frac{(-1)^n}{3} \right)$ b) $\pi \left(n + \frac{(-1)^n}{4} \right)$ c) $\pi \left(n + \frac{(-1)^n}{6} \right)$ d) $\frac{n\pi}{2}$
311. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b are respectively

- a) $-6, -12, -18$ b) $-6, 4, 9$ c) $-6, -4, -9$ d) $-6, 12, 18$
312. If A is a non-singular matrix of order 3, then $\text{adj}(\text{adj } A)$ is equal to
a) A b) A^{-1} c) $\frac{1}{|A|}A$ d) $|A|A$
313. If A is a matrix such that $A^2 = A + I$, where I is the unit matrix, then A^5 is equal to
a) $5A + I$ b) $5A + 2I$ c) $5A + 3I$ d) $5A + 4I$
314. If A and B are two matrices such that both $A+B$ and AB are defined, then
a) A and B are of same order b) A is of order $m \times m$ and B is of order $n \times n$
c) Both A and B are of same order $n \times n$ d) A is of order $m \times n$ and B is of order $n \times m$
315. If $U = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} 0 & 2 & 3 \end{bmatrix}$, $V = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$, then $UV + XY$
a) $[20]$ b) 20 c) $[-20]$ d) -20
316. The matrix $A = \begin{bmatrix} a & 2 \\ 2 & 4 \end{bmatrix}$ is singular, if
a) $a \neq 1$ b) $a = 1$ c) $a = 0$ d) $a = -1$
317. If $A = \begin{bmatrix} 1 & x \\ x^2 & 4y \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$, $\text{adj } A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,
Then values of x and y are
a) $1, 1$ b) $\pm 1, 1$ c) $1, 0$ d) None of these
318. $A = \begin{bmatrix} \cos \alpha - \sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is
a) A b) $-A$ c) $\text{adj } (A)$ d) $-\text{adj } (A)$
319. Which of the following is incorrect?
a) $A^2 - B^2 = (A + B)(A - B)$
b) $(A^T)^T = A$
c) $(AB)^n = A^n B^n$ where A, B commute
d) $(A - I)(I + A) = 0 \Leftrightarrow A^2 = I$
320. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = \frac{1+x}{1-x}$, then $f(A)$ is
a) $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ d) None of these
321. If A is an invertible matrix, then $\det(A^{-1})$ is equal to
a) $\det(A)$ b) $\frac{1}{\det(A)}$ c) 1 d) None of these
322. If A and B are square matrices of the same order and $AB = 3I$, then A^{-1} is equal to
a) $3B$ b) $\frac{1}{3}B$ c) $3B^{-1}$ d) $\frac{1}{3}B^{-1}$
323. If the points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are collinear,
then the rank of the matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ will always be less than
a) 2 b) 3 c) 1 d) None of these
324. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the unit matrix of order 2, then A^2 equals
a) $4A - 3I$ b) $3A - 4I$ c) $A - I$ d) $A + I$
325. If $A = \begin{bmatrix} -2 & 6 \\ -5 & 7 \end{bmatrix}$, then $\text{adj } A$ is
a) $\begin{bmatrix} 7 & -6 \\ 5 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & -6 \\ 5 & -7 \end{bmatrix}$ c) $\begin{bmatrix} 7 & -5 \\ 6 & -2 \end{bmatrix}$ d) None of these
326. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ then $A \cdot (\text{adj } A)$ is equal to
a) A b) $|A|$ c) $|A|I$ d) None of these
327. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $a, b \in N$. Then

- a) There exist more than one but finite number of B 's such that $AB = BA$
 b) There exist exactly one B such $AB = BA$
 c) There exists infinitely many B 's such that $AB = BA$
 d) There cannot exist any B such that $AB = BA$
328. If $2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$, then B is
 a) $\begin{bmatrix} 8 & -1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$ c) $\begin{bmatrix} 8 & 1 & -2 \\ -1 & 10 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 8 & 1 & 2 \\ 1 & 10 & 1 \end{bmatrix}$
329. If I is unit matrix of order 10, then the determinant of I is equal to
 a) 10 b) 1 c) $1/10$ d) 9
330. Let $A = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB|$ is equal to
 a) 80 b) 100 c) -110 d) 92
331. If $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ is the sum of a symmetric matrix B and skew-symmetric matrix C , then B is
 a) $\begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 5 & -2 \\ 2 & 2 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 6 & 6 & 7 \\ -6 & 2 & -5 \\ -7 & 5 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 6 & -2 \\ 2 & 0 & -2 \\ -2 & -2 & 0 \end{bmatrix}$
332. Let A be a non-singular square matrix. Then, $|\text{adj } A|$ is equal to
 a) $|A|^n$ b) $|A|^{n-1}$ c) $|A|^{n-2}$ d) None of these
333. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix}$ $(A + B)^2 = A^2 + B^2$. Then a and b are respectively
 a) 1, -1 b) 2, -3 c) -1, 1 d) 3, -2
334. For non-singular square matrices A, B and C of same order, $(AB^{-1}C)^{-1}$ is equal to
 a) $A^{-1}BC^{-1}$ b) $C^{-1}B^{-1}A^{-1}$ c) $CB^{-1}A^{-1}$ d) $C^{-1}BA^{-1}$
335. If $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$, then $A^{-1} =$
 a) $\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

1 (a)

Let $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$. Then,

$$A^2 = AA = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\therefore A^3 = A, A^4 = A^2 = I, A^2 = A^4 A = IA = A \text{ etc.}$$

$$\text{Hence, } A^n = \begin{cases} A, & \text{if } n \text{ is odd} \\ I, & \text{if } n \text{ is even} \end{cases}$$

2 (d)

$$\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} = \begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}$$

$$\text{and } \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2 \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2$$

$$= \begin{bmatrix} a^2 b^2 & 0 \\ 0 & a^2 b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [\text{given}]$$

$$\Rightarrow a^2 b^2 = 1 \Rightarrow ab = 1$$

3 (b)

Given matrix A is a square matrix

$$\text{And } AA^T = I = A^T A$$

$$\Rightarrow |AA^T| = |I| = |A^T A|$$

$$\Rightarrow |A||A^T| = 1 = |A^T||A|$$

$$\Rightarrow |A|^2 = 1 \quad [\because A \cdot A^T = |A|^2]$$

$$\Rightarrow |A| = \pm 1$$

4 (b)

$$\text{Let } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{ then, } kI = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

$$\Rightarrow \text{adj}(kI) = \begin{bmatrix} k^2 & 0 & 0 \\ 0 & k^2 & 0 \\ 0 & 0 & k^2 \end{bmatrix} = k^2 I$$

5 (b)

$$\text{Given, } A \text{ is singular} \Rightarrow |A| = 0$$

$$\text{Now, } A(\text{adj } A) = |A|I_n = 0$$

$$\therefore A(\text{adj } A) = 0 \text{ ie, } A(\text{adj } A) \text{ is a zero matrix.}$$

6 (c)

$$\text{Given, } AB = O$$

$$\therefore \begin{bmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha \end{bmatrix} \times \begin{bmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos \alpha \cos \beta \cos(\alpha - \beta) & \cos \alpha \sin \beta \cos(\alpha - \beta) \\ \cos \beta \sin \alpha \cos(\alpha - \beta) & \sin \alpha \sin \beta \cos(\alpha - \beta) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \cos(\alpha - \beta) = 0$$

$$\Rightarrow \alpha - \beta \text{ is an odd multiple of } \frac{\pi}{2}$$

8 (b)

$$\text{Given, } A = \begin{vmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ -6 & -12 & -15 \end{vmatrix}$$

$$\text{Applying } R_2 \rightarrow R_2 - 2R_1 \text{ and } R_3 \rightarrow R_3 + 3R_1$$

$$\Rightarrow A = \begin{vmatrix} 2 & 4 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

Since the equivalent matrix in echelon form has only one non-zero row,

$$\therefore \text{Rank}(A) = 1$$

9 (c)

$$\therefore AB = B, BA = A$$

$$\therefore A^2 + B^2 = AA + BB = A(BA) + B(AB)$$

$$= (AB)A + (BA)B = BA + AB$$

$$= A + B$$

$$\Rightarrow A^2 = A \text{ and } B^2 = B$$

10 (d)

$$|A| = 7(1 - 0) + 3(-1 - 0) - 3(0 + 1) = 1$$

Cofactors of matrix A are

$$C_{11} = 1, \quad C_{12} = 1, \quad C_{13} = 1$$

$$C_{21} = 3, \quad C_{22} = 4, \quad C_{23} = 3$$

$$C_{31} = 3, \quad C_{32} = 3, \quad C_{33} = 4$$

$$\therefore \text{adj}(A) = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

11 (d)

It is given that A is a skew-symmetric matrix

$$\therefore A^T = -A$$

$$\Rightarrow (A^T)^n = (-A)^n$$

$$\Rightarrow (A^n)^T = (-1)^n A^n$$

$$\Rightarrow (A^n)^T = \begin{cases} A^n & \text{if } n \text{ is even} \\ -A^n & \text{if } n \text{ is odd} \end{cases}$$

Hence, A^n is skew-symmetric when n is odd

12 (d)

$$\text{Consider } \begin{bmatrix} 1 & * & * \\ * & 1 & * \\ * & * & 1 \end{bmatrix}. \text{ By placing } a_1 \text{ in any one of the}$$

6*

Position and 0 elsewhere. We get 6 non-singular matrices.

$$\text{Similarly, } \begin{bmatrix} * & * & 1 \\ * & 1 & * \\ 1 & * & * \end{bmatrix} \text{ gives at least one non-singular.}$$

13 (a)

$$\text{We know that, } \text{Tr}(A) = \sum_{i=1}^3 a_{ii}$$

$$\text{Tr}(A) = 1 + 7 + 9 = 17$$

14 (d)

$$\text{Now, } A + B = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow |A + B| = \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 2 - 4 = -2$$

$$\text{Also, } \text{adj}(A + B) = \begin{bmatrix} 2 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\begin{aligned}\therefore (A+B)^{-1} &= -\frac{1}{2} \begin{bmatrix} 2 & -2 \\ -2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 1 \\ 1 & -1/2 \end{bmatrix} \\ A^{-1} &= \frac{1}{4} \begin{bmatrix} 4 & -4 \\ -2 & 3 \end{bmatrix} \text{ and } B^{-1} = \frac{1}{4} \begin{bmatrix} -2 & 2 \\ 0 & -2 \end{bmatrix} \\ \therefore A^{-1} + B^{-1} &= \begin{bmatrix} 1 & -1 \\ -1/2 & 3/4 \end{bmatrix} + \begin{bmatrix} -1/2 & 1/2 \\ 0 & -1/2 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/4 \end{bmatrix} \\ \therefore (A+B)^{-1} &\neq A^{-1} + B^{-1}\end{aligned}$$

15 (a)

$$\begin{aligned}f(A) &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}\end{aligned}$$

16 (b)

$$\begin{aligned}C_{11} &= 1, C_{12} = -2, C_{13} = -2 \\ C_{21} &= -1, C_{22} = 3, C_{23} = 3 \\ C_{31} &= 0, C_{32} = -4, C_{33} = -3 \\ \therefore \text{adj}(A) &= \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}\end{aligned}$$

17 (c)

Since, the rank of given matrix is 1, then

$$\begin{aligned}\begin{bmatrix} 2 & 5 \\ -4 & a-4 \end{bmatrix} &= 0 \\ \Rightarrow 2a - 8 + 20 &= 0 \\ \Rightarrow a &= -6\end{aligned}$$

18 (a)

Since, given equations have a non-trivial solution

$$\begin{aligned}\therefore \Delta &= 0 \\ \Rightarrow \begin{vmatrix} 1 & 3 & \lambda \\ 2 & 4 & -1 \\ 1 & 5 & -2 \end{vmatrix} &= 0 \\ \Rightarrow 1(-8+5) - 3(-4+1) + \lambda(10-4) &= 0 \\ \Rightarrow 6\lambda &= -6 \Rightarrow \lambda = -1\end{aligned}$$

20 (b)

Since, $|a| \neq 0$ So, A^{-1} exists

$$\begin{aligned}\therefore AB &= AC \\ \Rightarrow A^{-1}(AB) &= A^{-1}(AC) \\ \Rightarrow (A^{-1}A)B &= (A^{-1}A)C \Rightarrow B = C\end{aligned}$$

21 (c)

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix}, B = \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{aligned}\text{For consistent, } |A| &= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 0 \\ \Rightarrow 1(20-16) - 1(10-4) + 1(4-2) &= 0 \\ \Rightarrow 4-6+2 &= 0 \\ \Rightarrow 0 &= 0 \\ \text{and } (\text{adj } A)B &= 0\end{aligned}$$

$$\begin{aligned}\Rightarrow \begin{bmatrix} 4 & -6 & 2 \\ -6 & 9 & -3 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 4-6\alpha+2\alpha^2 \\ -6+9\alpha-3\alpha^2 \\ 2-3\alpha+\alpha^2 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\end{aligned}$$

$$\Rightarrow 2\alpha^2 - 6\alpha + 4 = 0, -3\alpha^2 + 9\alpha - 6 = 0$$

$$\text{and } \alpha^2 - 3\alpha + 2 = 0$$

$$\text{Now, } 2\alpha^2 - 6\alpha + 4 = 0$$

$$\Rightarrow (2\alpha-2)(\alpha-2) = 0$$

$$\Rightarrow \alpha = 1, 2$$

Similarly from other equations we also get the same value

22 (b)

Given, $B = A^6 - A^5$, where $A^3 = A + I$

$$\begin{aligned}\Rightarrow B &= (A^3)^2 - A^3 A^2 \\ &= (A + I)^2 - (A + I)A^2 \\ &= A^2 + I^2 + 2AI - A^3 - A^2 I \\ &= I + 2A - (A + I) \\ \Rightarrow B &= A\end{aligned}$$

$$\therefore \text{Inverse of } B = A^{-1}$$

23 (d)

Since, $A^T A = I$

$$\begin{aligned}\Rightarrow \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\end{aligned}$$

$$\Rightarrow a^2 + b^2 + c^2 = 1 \text{ and } ab + bc + ca = 0$$

$$\text{Now, } (a+b+c)^2 =$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$= 1 + 2 \cdot 0 = 1$$

$$\Rightarrow a + b + c = 1 \dots (i)$$

$$\text{Now, } (a^3 + b^3 + c^3) = (a + b + c)$$

$$(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$= (a + b + c) + 3$$

$$\Rightarrow a^3 + b^3 + c^3 = 1 + 3 = 4 \text{ [using Eq. (i)]}$$

24 (d)

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now,

$$A^2 - 4A = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 5I_3$$

25 (a)

We have,

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$\Rightarrow A^4 = A^2 A^2 = I_2 I_2 = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

26 (b)

The system of given equation are

$$Kx + 2y - z = 1 \quad \dots(i)$$

$$(K - 1)y - 2z = 2 \quad \dots(ii)$$

$$\text{and} \quad (K + 2)z = 3 \dots(iii)$$

This system of equations has a unique solution, if

$$\begin{vmatrix} K & 2 & -1 \\ 0 & K-1 & -2 \\ 0 & 0 & K+2 \end{vmatrix} \neq 0$$

$$\Rightarrow (K - 2)(K)(K - 1) \neq 0$$

$$\Rightarrow K \neq 2, 0, 1$$

ie, $K = -1$, is a required answer.

27 (d)

Given, $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$

$$\text{For no solution, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 3) + 1(2 - 2) = 0$$

$$\lambda - 3 = 0 \Rightarrow \lambda = 3$$

$$\text{and } \Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 3 \end{vmatrix} \neq 0$$

$$\Rightarrow 6(6 - 6) - 1(30 - 3\mu) + 1(20 - 2\mu) \neq 0$$

$$\Rightarrow \mu - 10 \neq 0 \Rightarrow \mu \neq 10$$

28 (b)

$$|AA^T| = |I| = |A^T A|$$

$$\Rightarrow |A||A^T| = 1 = |A^2||A|$$

$$\Rightarrow |A|^2 = 1 \quad [\because |A^T| = |A|]$$

$$\Rightarrow |A| = \pm 1$$

29 (c)

AB

$$= \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix} \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta \cos^2 \phi + \sin \theta \cos \phi \cos \theta \sin \phi & \cos^2 \phi \cos \theta \sin \theta + \sin^2 \theta \sin \phi \cos \phi \\ \cos^2 \theta \sin \phi \cos \phi + \sin^2 \theta \sin \theta \cos \theta & \cos \theta \sin \theta \sin \phi \cos \phi + \sin^2 \theta \sin^2 \phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta \cos \phi \cos(\theta - \phi) & \sin \phi \cos \theta \cos(\theta - \phi) \\ \sin \theta \cos \phi \cos(\theta - \phi) & \sin \theta \sin \phi \cos(\theta - \phi) \end{bmatrix}$$

$$\therefore AB = O$$

$$\Rightarrow \cos(\theta - \phi) = 0$$

$$\Rightarrow \cos(\theta - \phi) = \cos(2n + 1)\frac{\pi}{2}$$

$$\Rightarrow \theta = (2n + 1)\frac{\pi}{2} + \phi, \text{ where } n = 0, 1, 2, \dots$$

30 (a)

We have,

$$AB = A \text{ and } BA = B$$

$$\text{Now, } AB = A$$

$$\Rightarrow (AB)A = AA$$

$$\Rightarrow A(BA) = A^2$$

$$\Rightarrow AB = A^2 \quad [\because BA = B]$$

$$\Rightarrow A = A^2 \quad [\because AB = A]$$

$$\text{and, } BA = B$$

$$\Rightarrow (BA)B = B^2$$

$$\Rightarrow B(AB) = B^2$$

$$\Rightarrow BA = B^2 \quad [\because AB = A]$$

$$\Rightarrow B = B^2 \quad [\because BA = B]$$

$$\therefore A^2 = A \text{ and } B^2 = B$$

$$\Rightarrow A \text{ and } B \text{ are idempotent matrices}$$

31 (b)

From given matrix equation, we have

$$x - y - z = 0$$

$$-y + z = 5$$

$$z = 3$$

$$\Rightarrow x = 1, y = -2, z = 3$$

32 (d)

Here, cofactors are

$$C_{11} = \cos 2\theta, \quad C_{12} = -\sin 2\theta$$

$$C_{21} = \sin 2\theta, \quad C_{22} = \cos 2\theta$$

$$\therefore |A| = |\cos^2 2\theta + \sin^2 2\theta| = 1$$

$$\therefore A^{-1} = \frac{1}{|A|} \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

33 (c)

Since, B is the inverse of A .

$$\text{ie, } B = A^{-1}$$

$$\text{So, } 10A^{-1} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow 10A^{-1}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} A$$

$$= \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -5 + \alpha & 5 + \alpha & -5 + \alpha \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow -5 + \alpha = 0 \Rightarrow \alpha = 5$$

34 (a)

$$AB = \begin{bmatrix} 0 & 3 & 3 \\ -3 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3y + 3z \\ -3x - 4z \\ -3x + 4y \end{bmatrix}$$

$$\text{Now, } B'(AB) = [x \ y \ z] \begin{bmatrix} 3y + 3z \\ -3x - 4z \\ -3x + 4y \end{bmatrix}$$

$$= [3xy + 3zx - 3xy - 4yz - 3xz + 4yz]$$

$$= [0]$$

$$\therefore B'(AB) \text{ is a null matrix.}$$

36 (a)

$$\text{Given, } A = \begin{bmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow |A| = a^2 + b^2 = 1$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj}(A) = \text{adj}(A)$$

38 (c)

We have,

$$(A + B)^2 = A^2 + 2AB + B^2$$

$$\Leftrightarrow (A + B)(A + B) = A^2 + 2AB + B^2$$

$$\Leftrightarrow A^2 + AB + BA + B^2 = A^2 + 2AB + B^2$$

$$\Leftrightarrow AB = BA$$

$$\text{Hence, } (A + B)^2 = A^2 + 2AB + B^2 \Leftrightarrow AB = BA$$

39 (d)

$$A^{-1} = \frac{1}{2x^2} \begin{bmatrix} x & 0 \\ -x & 2x \end{bmatrix} = \begin{bmatrix} \frac{1}{2x} & 0 \\ -\frac{1}{2x} & 1 \end{bmatrix}$$

$$\text{But } A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{1}{2x} & 0 \\ -\frac{1}{2x} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \Rightarrow \frac{1}{2x} = 1 \Rightarrow x = \frac{1}{2}$$

40 (c)

Given,

$$\therefore \begin{vmatrix} a & a-1 \\ b-1 & b \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$

$$\Rightarrow \begin{vmatrix} a & 0 & -(1+a) \\ b & -(1+b) & 0 \\ -1 & c+1 & c+1 \end{vmatrix} = 0$$

$$\Rightarrow -a(1+b)(1+c) - b[0 + (1+a)(1+c)]$$

$$-1[0 - (1+a)(1+b)] = 0$$

$$\Rightarrow -a(1+b)(1+c) - b(1+a)(1+c)$$

$$+1(1+a)(1+b) = 0$$

On dividing by $(1+a)(1+b)(1+c)$, we get

$$-\frac{a}{1+a} - \frac{b}{1+b} + \frac{1}{1+c} = 0$$

$$\Rightarrow -\frac{a}{1+a} + 1 - \frac{b}{1+b} + 1 + \frac{1}{1+c} = 2$$

$$\Rightarrow \frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = 2$$

41 (a)

The given system of equation has at least one solution, if

$$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 3(2\lambda + 15) + 1(\lambda + 18) + 4(5 - 12) = 0$$

$$\Rightarrow 7\lambda = -35 \Rightarrow \lambda = -5$$

43 (d)

$$\text{Given, } A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$C_{11} = 4, C_{12} = 1, C_{13} = -2$$

$$C_{21} = -2, C_{22} = 4, C_{23} = 1$$

$$C_{31} = 1, C_{32} = -2, C_{33} = 4$$

$$\therefore \text{adj}(A) = \begin{bmatrix} 4 & 1 & -2 \\ -2 & 4 & 1 \\ 1 & -2 & 4 \end{bmatrix}^T = \begin{bmatrix} 4 & -2 & 1 \\ 1 & 4 & -2 \\ -2 & 1 & 4 \end{bmatrix}$$

$$\therefore |\text{adj } A| = 4(16 + 2) + 2(4 - 4) + (1 + 8) = 72 + 0 + 9 = 81$$

Alternate

$$|A| \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix}$$

$$= 2(4 - 0) - 1(0 - 1) = 9$$

$$\therefore |\text{adj } A| = |A|^{3-1} = (9)^2 = 81$$

44 (a)

Since, $A' = -A$

$$\therefore A^3 = AAA$$

$$\text{And } (A^3)' = A'A'A' = -A^3$$

Hence, matrix A^3 is a skew-symmetric matrix

45 (c)

$$\text{Given, } [1 \ x \ 1] \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$$

$$\Rightarrow [1 \ x \ 1] \begin{bmatrix} x+2-6 \\ 0+5-2 \\ 0+3-4 \end{bmatrix} = 0$$

$$\Rightarrow [1 \ x \ 1] \begin{bmatrix} x-4 \\ 3 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow x - 4 + 3x - 1 = 0 \Rightarrow x = \frac{5}{4}$$

46 (d)

$$\text{We have, } A + b = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

Clearly, $A + B$ is a symmetric matrix

Now,

$$(A + B)^{-1} = -1 \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 4 & -4 \\ -2 & 3 \end{bmatrix}, B^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix}$$

$$\therefore A^{-1} + B^{-1} = \frac{1}{4} \begin{bmatrix} 2 & 0 \\ -2 & -1 \end{bmatrix}$$

We observe that $(A + b)^{-1}$ is a symmetric matrix and $(A + B)^{-1} \neq A^{-1} + B^{-1}$

47 (c)

$$\therefore A \cdot A^T = I_n$$

$$\Rightarrow A - I_n = A - AA^T = A(I_n - A^T)$$

$$\Rightarrow |A - I_n| = |A(I_n - A^T)|$$

$$= |A||I_n - A^T|$$

$$= |A||I_n - A|$$

48 (c)

$$A^2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} - \frac{1}{2} & \frac{1}{2} - \frac{1}{2} & \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} & \frac{1}{2} - \frac{1}{2} & \frac{1}{2} - \frac{1}{2} \\ -\frac{1}{2} + \frac{1}{2} & -\frac{1}{2} + \frac{1}{2} & -\frac{1}{2} + \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ Matrix A is nilpotent of order 2.

49 (a)

Let A be a symmetric matrix. Then,

$$A A^{-1} = I$$

$$\Rightarrow (A A^{-1})^T = I$$

$$\Rightarrow (A^{-1})^T A^T = I$$

$$\Rightarrow (A^{-1})^T = (A^T)^{-1}$$

$$\Rightarrow (A^{-1})^T = (A)^{-1} [\because A^T = A]$$

$\Rightarrow A^{-1}$ is a symmetric matrix

50 (a)

Since, given matrix is a triangular matrix, so its characteristic roots are the diagonal elements.

Hence, required roots are 1, 3, 6.

51 (d)

$$\text{Given that, } A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\text{and } A^{-1} = \frac{1}{k} \text{adj } A$$

$$\Rightarrow k = |A|$$

$$= \begin{vmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 3(2 + 1) - 2(1 - 0) + 4(1 - 0)$$

$$= 9 - 2 + 4 = 11$$

52 (a)

Since, $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is linear equation in three

variables and that could have only unique, no solution or infinitely many solution.

$\therefore$ It is not possible to have two solutions.

Hence, number of matrices A is zero.

53 (c)

Since, A is invertible matrix of order n , then the determinant of $\text{adj } A = |A|^{n-1}$

54 (c)

$$\text{Given, } \text{adj} \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix} \dots (i)$$

Cofactors of $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix}$ are

$$C_{11} = 5, C_{12} = 1, C_{13} = -2$$

$$C_{21} = 4, C_{22} = 1, C_{23} = -2$$

$$C_{31} = -2, C_{32} = 0, C_{33} = 1$$

$$\Rightarrow \begin{bmatrix} 5 & 4 & -2 \\ 1 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix}$$

On comparing the corresponding elements, we get

$$a = 4, \quad b = 1$$

$$\therefore [a \ b] = [4 \ 1]$$

55 (d)

$$AB = [x \ y \ z] \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

$$= [xa + yh + zg \quad xh + yb + zf \quad xg + yf + zc]$$

$$\text{Now, } ABC = O$$

$$\Rightarrow [xa + yh + zg \quad xh + yb + zf \quad xg + yf$$

$$+ zc] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = O$$

$$\Rightarrow [ax^2 + hxy + gxz + hxy + y^2b + fzy + gxz + yfz + z^2c]$$

$$= O$$

$$\Rightarrow [ax^2 + by^2 + cz^2 + 2gzx + 2hxy + 2fyz] = O$$

56 (d)

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Let B be the multiplicative inverse of A , then

$$AB = I$$

$$\Rightarrow B = A^{-1}$$

$$= \frac{1}{8-7} \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

57 (c)

$$\text{Let } A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 6 & 0 & 2 & 0 \end{bmatrix}$$

Now, we take a submatrix of order 3×3

$$B = \begin{bmatrix} 4 & 1 & 0 \\ 3 & 0 & 1 \\ 6 & 0 & 2 \end{bmatrix}$$

$$|B| = -1(6 - 6) = 0$$

Now, we take a submatrix of order 2×2 .

$$\text{ie, } C = \begin{bmatrix} 4 & 1 \\ 3 & 0 \end{bmatrix}$$

$$|C| = 0 - 3 \neq 0$$

$\therefore$ Rank of matrix A is 2.

58 (d)

$$\text{Let } A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B$$

$$= \frac{1}{2} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

59 (a)

$$\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 21 + 4 + 10 \\ 27 + 8 + 5 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 43 \\ 44 \end{bmatrix}$$

60 (b)

Since, A, B and C are $n \times n$ matrices and, if $A^3 + 2A^2 + 3A + 5I = 0$, then A is invertible.

61 (a)

We have,

$$|A| = -1, |B| = 3$$

$$\therefore |3AB| = 3^3|AB| = 3^3|A||B| = 3 \times -1 \times 3 = -9$$

62 (c)

$$\text{Let } A = \begin{bmatrix} 1230 \\ 2432 \\ 3213 \\ 6875 \end{bmatrix}. \text{ Then,}$$

$$A \sim \begin{bmatrix} 1230 \\ 2432 \\ 3213 \\ 0000 \end{bmatrix} \text{ By applying } R_4 \rightarrow R_4 - R_3 - R_2 - R_1$$

$$\Rightarrow A \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -32 & \\ 0 & -4 & -83 & \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ By applying } R_2 \rightarrow R_2 - 2R_1, \\ R_3 \rightarrow R_3 - 3R_1$$

We observe that the leading minor of the third order of this matrix is non-zero i.e.

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & -3 \\ 0 & -4 & -8 \end{vmatrix} = -12 \neq 0. \text{ Hence, rank}(A) = 3$$

63 (a)

$$\text{Given, } A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$$

$$\therefore A - I = A + I - 2I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ = \begin{bmatrix} 1 & -2 \\ 4 & -1 \end{bmatrix}$$

$$\therefore (A + I)(A - I) = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$$

64 (d)

We have,

$$A = kI_n \Rightarrow |A| = |kI_n| = k^n|I_n| = k^n$$

65 (d)

$$\text{Given, } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + dc & bc + d^2 \end{bmatrix}$$

$$\therefore A^2 - (a + d)A + kI = 0$$

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + dc & bc + d^2 \end{bmatrix} - \begin{bmatrix} a^2 + ad & ab + bd \\ ac + dc & ad + d^2 \end{bmatrix} \\ + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} bc - ad + k & 0 \\ 0 & bc - ad + k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On equating, we get

$$bc - ad + k = 0$$

$$\Rightarrow k = ad - bc \quad \dots(i)$$

$$\text{Also, } \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$$

$$\Rightarrow ad - bc = 0$$

$$\therefore \text{From Eq. (i), } k = 0$$

66 (b)

$$\text{Given that, } A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - 6 + 1 & 1 - 4 + 1 \\ 4 + 3 + 3 & 2 + 2 + 3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$$

$$\Rightarrow (AB)^T = \begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$$

68 (b)

We have,

$$A^2 = AA = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$\therefore A^2 - 5A = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5 \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 - 5A = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} = 14I$$

71 (b)

Given that A is a singular matrix

$$\therefore |A| = 0$$

$$\therefore A \text{ adj } A = |A| = 0$$

$$\therefore A \text{ adj } A \text{ is a zero matrix}$$

72 (a)

$$\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \frac{1}{-1} \begin{bmatrix} 1 & -1 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$$

73 (b)

$$\text{Given, } A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 6 & 2 \\ 3 & 4 & 1 \end{bmatrix} \text{ and } A^3 = \begin{bmatrix} 7 & 9 & 3 \\ 15 & 19 & 6 \\ 9 & 12 & 4 \end{bmatrix}$$

$$\text{Hence, } A^3 - 3A^2 - I = 0$$

74 (d)

$$\therefore PQ = I \Rightarrow P^{-1} = Q$$

Now, the system in matrix notation is $PX = B$

$$\therefore X = P^{-1}B = QB$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 2 & 2 & 1 \\ 13 & -5 & m \\ -8 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

$$\Rightarrow y = \frac{1}{9}(13 - 5 + 5m)$$

$$\Rightarrow -27 = 8 + 5m \quad (\text{given } y = -3)$$

$$\therefore m = -7$$

75 **(d)**

Since A is invertible. Therefore, $|A| \neq 0$

Thus, option (d) is correct.

76 **(a)**

We have,

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1} \right)$$

$$\Rightarrow A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \right)$$

$$\Rightarrow A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

77 **(c)**

If A is a square matrix, then $A - A^T$ is a skew-symmetric matrix, then $|A - A^T|$ is '0' or a perfect square as A is of odd order or even order

78 **(d)**

$$O(A') = 3 \times 2, O(B') = 2 \times 3$$

$$(a) CB + A'$$

Now, order of CB

$$= (\text{order of } C \text{ is } 3 \times 3)(\text{order of } B \text{ is } 3 \times 2)$$

$$= \text{order of } CB \text{ is } 3 \times 2$$

$$\text{Since, } O(A') = 3 \times 2$$

$\therefore$ Matrix $CB + A'$ can be determined.

$$(b) O(BA) = 3 \times 3$$

$$\text{and } O(C) = 3 \times 3$$

$\therefore$ Matrix BAC can be determined.

$$(c) O(A + B') = 2 \times 3$$

$$\Rightarrow O(A + B')' = 3 \times 2$$

$$\text{and } O(C) = 3 \times 3$$

$\therefore$ Matrix $C(A + B')$ can be determined.

$$(d) O(A + B') = 2 \times 3$$

$$\text{And } O(C) = 3 \times 3$$

$\therefore$ Matrix $C(A + B')$ cannot be determined.

79 **(d)**

Given A is a matrix of order 3 and $B =$

$$|A|A^{-1}, |A| = -5$$

$$\therefore B = |A| \frac{(\text{adj } A)}{|A|} \Rightarrow B = (\text{adj } A)$$

$$\Rightarrow |B| = |A|^{3-1} = 25$$

80 **(a)**

$$\therefore A^2 = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore A^2$ is null matrix.

81 **(b)**

For unique solution,

$$\begin{vmatrix} k & 2 & -1 \\ 0 & k-1 & -2 \\ 0 & 0 & k+2 \end{vmatrix} \neq 0$$

$$\Rightarrow k(k-1)(k+2) \neq 0$$

$$\Rightarrow k \neq 0, 1 \text{ or } -2$$

82 **(b)**

$$\text{Given, } A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\therefore A^{-1} = -\frac{1}{3} \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\text{Also, } AX = B$$

$$\Rightarrow X = A^{-1}B = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 3+2 \\ 6+1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

83 **(a)**

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix}$$

$$= 1(9 - 15) - 2(18 - 15) + 3(10 - 5)$$

$$= -6 - 6 + 15$$

$$= 3 \neq 0$$

Hence, the system of equations has a unique solution.

84 **(b)**

It is given that

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2A$$

$$\Rightarrow A^3 = 2(AA) = 2A^2 = 2(2A) = 2^2A$$

Continuing in this manner, we have $A^n = 2^{n-1}A$

85 **(a)**

We have,

$$(B^{-1}A^{-1})^{-1} = (A^{-1})^{-1}(B^{-1})^{-1} [\because (PQ)^{-1} = Q^{-1}P^{-1}]$$

$$\Rightarrow (B^{-1}A^{-1})^{-1} = AB$$

$$\Rightarrow (B^{-1}A^{-1}) = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow (B^{-1}A^{-1})^{-1} = \begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$$

Hence, option (a) is correct

86 **(c)**

Here, C , A and C^T are matrix of order $n \times 1$, $n \times n$ and $1 \times n$ respectively.

$$\text{Let } C^TAC = k$$

$$\text{Then, } (C^TAC)^T = C^TA^T(C^T)^T$$

$$= C^TA^TC = C^T(-A)C$$

$$= -C^TAC = -k$$

$$\Rightarrow k = -k \Rightarrow k = 0$$

$$\Rightarrow C^TAC \text{ is null matrix.}$$

Which shows that C^TAC is a zero matrix of order 1.

87 **(b)**

Since, $A + A^T$ is a square matrix

$$\therefore (A + A^T)^T = A^T + (A^T)^T = A^T + A$$

Hence, $A + A^T$ is symmetric matrix

88 (d)

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 2+2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$$

89 (a)

$$\text{Given, } P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 4 & 5 & 4 \\ 8 & 10 & 12 \\ 0 & 0 & -4 \end{bmatrix}$$

$$\therefore P_{22} = [2 \ 3 \ 4] \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$$

$$= 10 + 30 = 40$$

91 (c)

$$\text{Given, } AB = A \Rightarrow B = I$$

$$\text{And } BA = B \Rightarrow A = I$$

$$\therefore B = I \text{ and } A = I$$

$$\Rightarrow B^2 = B \text{ and } A^2 = A$$

92 (a)

$$\text{Let } A = \begin{bmatrix} 0 & x & y & z \\ -x & 0 & a & b \\ -y & -a & 0 & c \\ -z & -b & -c & 0 \end{bmatrix} \text{ be a skew-symmetric matrix.}$$

Then,

$$|A| = \begin{vmatrix} 0 & x & y & z \\ -x & 0 & a & b \\ -y & -a & 0 & c \\ -z & -b & -c & 0 \end{vmatrix} = (cx - by + az)^2$$

93 (d)

$$\text{Since, } A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\therefore B = \text{adj } A = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$$

$$\Rightarrow \text{adj } B = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix}$$

$$\Rightarrow |\text{adj } B| = \begin{vmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{vmatrix} = 625$$

Given that, $C = 5A$

$$\Rightarrow |C| = 5^3 |A| = 125 \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix} = 625$$

$$\text{Hence, } \frac{|\text{adj}(B)|}{|C|} = \frac{625}{625} = 1$$

94 (a)

$$\text{Given, } A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$$

$$\text{and symmetric matrix } B = \frac{A+A'}{2}$$

$$\therefore B = \frac{1}{2} \left\{ \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix} \right\} = \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$$

95 (d)

$$\text{The matrix } \begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix} \text{ is singular, if}$$

$$\begin{vmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -2 & -2 & b \end{vmatrix} = 0$$

$$\Rightarrow -1(60 + 12) + 2(30 + 6) + b(-20 + 20) = 0$$

$$\Rightarrow -72 + 72 + 0b = 0$$

Hence, the given matrix is singular for any value of b

96 (b)

$$\det(2AB) = 2^3 \det(A) \det(B)$$

$$= 8 \det(A) \det(B)$$

97 (b)

$$c_{ij} = \sum_{k=1}^n a_{ik} b_{kj} \text{ (In general)}$$

And in a diagonal matrix non-diagonal elements are zero *ie*,

$$a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ a_{ij}, & \text{if } i = j \end{cases}$$

$$\text{So, } c_{ij} = a_{ii} b_{ij}$$

98 (b)

$$\text{Here, } |A| = (1)(9) - 2(-11) - 3(6)$$

$$= 9 + 22 - 18 = 33$$

$$\text{Since, } A^{-1} \text{adj}(A^{-1}) = |A^{-1}| I_3$$

$$\Rightarrow A^{-1} \text{adj}(A^{-1}) = (|A|^{-1}) I_3$$

$$\Rightarrow A \cdot A^{-1} \text{adj}(A^{-1}) = (|A|)^{-1} A I_3$$

$$\Rightarrow \text{adj}(A^{-1}) = (|A|)^{-1} A$$

$$\Rightarrow |A| \text{adj}(A^{-1}) = A \text{ (But } |A| \neq 0)$$

99 (d)

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\therefore |A| = 1(1+1) + 1(1-1) + 1(1+1) = 4 \neq 0$$

$$\therefore \text{Rank of matrix } A \text{ is } 3$$

100 (b)

Let $\frac{x^2}{a^2} = X$, $\frac{y^2}{b^2} = Y$ and $\frac{z^2}{c^2} = Z$, then given equation will be

$$X + Y - Z = 1, X - Y + Z = 1, -X + Y + Z = 1$$

$$\text{Here, } A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\text{Now, } |A| = -4 \neq 0$$

Therefore, the given system of equation has unique solution.

102 (b)

$$\left[\frac{1}{2} (A - A') \right]' = \frac{1}{2} (A - A')' = \frac{1}{2} (A' - A)$$

$$= -\frac{1}{2} (A - A')$$

Hence, it is a skew-symmetric matrix.

103 (b)

$$\therefore \text{adj } A = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

$$\text{and } |A| = \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = 10$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

105 (b)

$$\text{Let } A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

106 (d)

We have,

$$3A^3 + 2A^2 + 5A + I = 0$$

$$\Rightarrow I = -3A^3 - 2A^2 - 5A$$

$$\Rightarrow I A^{-1} = (-3A^3 - 2A^2 - 5A)A^{-1}$$

$$\Rightarrow A^{-1} = -3A^2 - 2A - 5I$$

108 (b)

The given system of equations are

$$x + y + z = 0,$$

$$2x + 3y + z = 0 \text{ and } x + 2y = 0$$

$$\text{Here, } \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 1(0 - 2) - 1(0 - 1) + (4 - 3)$$

$$= -2 + 1 + 1 = 0$$

$\therefore$ This system has infinite solutions

109 (c)

$$2X - \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 4 & 4 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 & 2 \\ 7/2 & 1 \end{bmatrix}$$

111 (b)

$$\text{Given, } A = A', B = B'$$

$$\text{Now, } (AB - BA)' = (AB)' - (BA)'$$

$$= B'A' - A'B'$$

$$= BA - AB$$

$$= -(AB - BA)$$

$\therefore AB - BA$ is a skew-symmetric matrix.

113 (a)

Given that, p is a non-singular matrix such that

$$1 + p + p^2 + \dots + p^n = 0$$

$$\Rightarrow (1 + p)(1 + p + p^2 + \dots + p^{n-1}) = 0$$

$$\Rightarrow 1 - p^{n+1} = 0$$

$$\Rightarrow p^{n+1} = 1$$

$$\Rightarrow p^n \times p^1 = 1$$

$$\Rightarrow p^n = 1/p$$

$$\therefore p^{-1} = p^n$$

114 (a)

$$\text{Given, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 25 + 0 - 25 \\ -25 + 0 + 65 \\ 50 + 0 + 30 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 0 \\ 40 \\ 80 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow x = 0, y = 1, z = 2$$

$$\therefore x + y + z = 0 + 1 + 2 = 3$$

115 (d)

$$\therefore \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

Then, $X = y$ and $Y = x$

ie, $y = x$

116 (c)

$$\text{Given } \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} =$$

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \cdot \frac{1}{1 + \tan^2 \theta} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \frac{1}{1 + \tan^2 \theta} \begin{bmatrix} 1 - \tan^2 \theta & -2 \tan \theta \\ 2 \tan \theta & 1 - \tan^2 \theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} & \frac{2 \tan \theta}{1 + \tan^2 \theta} \\ \frac{2 \tan \theta}{1 + \tan^2 \theta} & \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow a = \cos 2\theta, \quad b = \sin 2\theta$$

117 (d)

$$\text{Given, } A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}$$

$$\Rightarrow \text{adj } A = (B)' = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = 3A'$$

118 (d)

Given equations are

$$px + y + z = 0, x + qy + z = 0, x + y + rz = 0$$

Since, the system have a non-zero solution, then

$$\begin{bmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{bmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_2$

$$\Rightarrow \begin{bmatrix} p & 1-p & 0 \\ 1 & q-1 & 1-q \\ 1 & 0 & r-1 \end{bmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r) \begin{bmatrix} \frac{p}{1-p} & 1 & 0 \\ \frac{1}{1-q} & -1 & 1 \\ \frac{1}{1-r} & 0 & -1 \end{bmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r) \left[\frac{p}{1-p}(1) - 1 \left(-\frac{1}{1-q} - \frac{1}{1-r} \right) \right] = 0$$

Since, $p, q, r \neq 1$

$$\therefore \frac{p}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \frac{1}{1-p} - 1 + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

119 (b)

Given, $AB = I \Rightarrow B = A^{-1}$

Now, $A^{-1} = \frac{\text{adj } A}{|A|}$

$$\begin{aligned} &= \frac{\begin{bmatrix} 1 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 1 \end{bmatrix}}{1 + \tan^2 \frac{\theta}{2}} \\ &= \frac{A^T}{\sec^2 \frac{\theta}{2}} = \cos^2 \frac{\theta}{2} A^T \end{aligned}$$

120 (d)

Given equations are

$$3x + y + 2z = 3 \quad \dots(i)$$

$$2x - 3y - z = -3 \quad \dots(ii)$$

$$\text{and } x + 2y + z = 4 \quad \dots(iii)$$

$$\text{Let } A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{aligned} \therefore |A| &= \begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix} \\ &= 3 \begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} \\ &= 3(-3+2) - 1(2+1) + 2(4+3) \\ &= -3 - 3 + 14 = 8 \end{aligned}$$

$$\text{adj. } A = \begin{bmatrix} -1 & -3 & 7 \\ 3 & 1 & -5 \\ 5 & 7 & -11 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\begin{aligned} &= \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix} \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix} \\ &= \frac{1}{8} \begin{bmatrix} -3-9+20 \\ -9-3+28 \\ 21+15-44 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 8 \\ 16 \\ -8 \end{bmatrix} \\ &\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \end{aligned}$$

$$\Rightarrow x = 1, y = 2, z = -1$$

121 (a)

$$A^2 = 2A - I$$

$$\therefore A^2 A = 2AA - IA$$

$$= 2A^2 - A = 2(2A - I) - A$$

$$\Rightarrow A^3 = 3A - 2I$$

$$\Rightarrow A^3 \cdot A = 3AA - 2IA = 3(2A - I) - 2A$$

$$\Rightarrow A^4 = 4A - 3I$$

$$\text{Similarly, } A^n = nA - (n-1)I$$

122 (d)

$$\det(M_r) = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix} = 2r - 1$$

$$\begin{aligned} \sum_{r=1}^{2007} \det(M_r) &= 2 \sum_{r=1}^{2007} r - 2007 \\ &= 2 \times \frac{2007 \times 2008}{2} - 2007 = (2007)^2 \end{aligned}$$

123 (a)

$$\text{Let } A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix}$$

$$\begin{aligned} \therefore |A| &= \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{vmatrix} \\ &= 0 \begin{vmatrix} 0 & 2 \\ -2 & 0 \end{vmatrix} - 1 \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} - 1 \begin{vmatrix} -1 & 0 \\ 1 & -2 \end{vmatrix} \\ &= 0 + 2 - 2 = 0 \\ &\Rightarrow |A| = 0 \end{aligned}$$

$$\begin{aligned} \text{Now, } (\text{adj } A)B &= \begin{bmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 4-4+6 \\ 2-2+3 \\ 2-2-3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -3 \end{bmatrix} \neq 0 \end{aligned}$$

$\therefore$ This system of equation is inconsistent, so it has no solution

125 (c)

Given, $D = \text{diag}(d_1, d_2, d_3, \dots, d_n)$

$$\Rightarrow D^{-1} = \text{diag}(d_1^{-1}, d_2^{-1}, \dots, d_n^{-1})$$

126 (a)

We have,

$$a = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix} \quad [\text{Using PMI}]$$

$$\Rightarrow \frac{1}{n} A^n = \begin{bmatrix} \frac{1}{n} & a \\ 0 & \frac{1}{n} \end{bmatrix} \Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} A^n = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$$

127 (d)

The given system of equations are

$$2x + y - 5 = 0 \quad \dots(i)$$

$$x - 2y + 1 = 0 \quad \dots(ii)$$

$$\text{and } 2x - 14y - a = 0 \quad \dots(iii)$$

This system is consistent.

$$\therefore \begin{vmatrix} 2 & 1 & -5 \\ 1 & -2 & 1 \\ 2 & -14 & -a \end{vmatrix} = 0$$

$$\Rightarrow 2(2a + 14) - 1(-a - 2) - 5(-14 + 4) = 0$$

$$\Rightarrow 4a + 28 + a + 2 + 50 = 0$$

$$\Rightarrow 5a = -80 \Rightarrow a = -16$$

128 (d)

The system of given equations has no solution,

$$\text{if } \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$ and taking common $(\alpha + 2)$ from C_1 , we get

$$(\alpha + 2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\Rightarrow (\alpha + 2) \begin{vmatrix} 1 & 1 & 1 \\ 0 & \alpha - 1 & 0 \\ 0 & 0 & \alpha - 1 \end{vmatrix} = 0$$

$$\Rightarrow (\alpha + 2)(\alpha - 1)^2 = 0$$

$$\Rightarrow \alpha = 1, -2$$

But $\alpha = 1$ makes given three equations same. So, the system of equation have infinite solution. So, answer is $\alpha = -2$ for which the system of equations has no solution

130 (b)

$$\text{Given, } A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow x^2 + 1 = 1, x = 0$$

$$\Rightarrow x = 0$$

131 (a)

Given that, $A^{-1} = \lambda (\text{adj } A)$

On comparing with $A^{-1} = \frac{1}{|A|\text{adj}} A$ we get

$$\lambda = \frac{1}{|A|}$$

$$\text{Now, } |A| = \begin{vmatrix} 0 & 3 \\ 2 & 0 \end{vmatrix} = 0 - 6 = -6$$

$$\Rightarrow \lambda = -\frac{1}{6}$$

132 (d)

$$a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14} = |A|$$

133 (d)

Given equation are $x + y + z = 6, x + 2y + 3z = 10$ and $x + 2y + \lambda z = 10$

Since, it is consistent.

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 3) + 1(2 - 2) = 0$$

$$\Rightarrow \lambda - 3 = 0 \Rightarrow \lambda = 3$$

134 (b)

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$$

$$\therefore A^4 = 2A \cdot 2A = 4A^2 = 4 \times 2A = 2^3 A$$

Similarly, $A^8 = 2^7 A$

$$\Rightarrow A^{100} = 2^{99} A$$

135 (d)

$$\text{Let } A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\therefore |A| = \cos^2 2\theta + \sin^2 2\theta = 1$$

$$\text{and } \text{adj } A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{1} \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

138 (a)

Give equation can be written as,

$$2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

139 (c)

We have,

$$A B = 0$$

$$\Rightarrow |A B| = 0$$

$$\Rightarrow |A||B| = 0$$

$$\Rightarrow |A| = 0 \text{ or } |B| = 0$$

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Then, $A B = O$. But

$$A \neq O, B \neq O$$

140 (b)

$$\text{Given, } A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 1 \\ 4 & -1 & -2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{3} \begin{bmatrix} -1 & 0 & 1 \\ 8 & 6 & -5 \\ -6 & -3 & 3 \end{bmatrix}$$

$$\text{Now, } A^{-1}D = \frac{1}{3} \begin{bmatrix} -1 & 0 & 1 \\ 8 & 6 & -5 \\ -6 & -3 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 8 \\ -1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8/3 \\ -1/3 \\ 0 \end{bmatrix}$$

142 (a)

Given equations are $x - cy - bz = 0$
 $cx - y + az = 0$ and $bx + ay - z = 0$

For non-zero solution

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$$

143 (b)

We have,

$$\text{Det}(I_n) = 1 (\neq 0) \Rightarrow \text{rank}(I_n) = n$$

144 (a)

The given matrix A is singular, if

$$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 8(7\lambda - 16) + 6(-6\lambda + 8) + 2(24 - 14) = 0$$

$$\Rightarrow 56\lambda - 128 - 36\lambda + 48 + 20 = 0$$

$$\Rightarrow 20\lambda = 60$$

$$\Rightarrow \lambda = 3$$

145 (c)

$$\text{Let } B = I + A + A^2 + A^3 \dots \infty$$

$$\Rightarrow AB = A + A^2 + A^3 + \dots \infty$$

$$\Rightarrow B - AB = I$$

$$\Rightarrow B(I - A) = I$$

$$\Rightarrow B = (I - A)^{-1}$$

$$\Rightarrow B = \begin{bmatrix} 0 & -2 \\ -3 & -3 \end{bmatrix}^{-1} = -\frac{1}{6} \begin{bmatrix} -3 & 2 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -1/3 \\ -1/2 & 0 \end{bmatrix}$$

146 (a)

Since A is non-singular matrix

$$\therefore |A| \neq 0 \Rightarrow \text{rank}(A) = n$$

147 (b)

$$A^2 = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix}$$

$$\text{Now, } f(A) = A^2 - 3A + 7$$

$$= \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix} - 3 \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$$

$$\therefore f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

148 (a)

The given system of equations will have a unique solution, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0$$

149 (a)

$$\text{Given, } 2x + y - z = 7 \quad \dots(i)$$

$$x - 3y + 2z = 1 \quad \dots(ii)$$

$$\text{and } x + 4y - 3z = 5 \quad \dots(iii)$$

From Eqs.(i) and (ii), we get

$$5x - y = 15 \quad \dots(iv)$$

From Eqs. (i) and (iii)

$$5x - y = 16 \quad \dots(v)$$

Eqs. (iv) and (v) shows that they are parallel and solution does not exist.

150 (d)

We have,

$$X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow X^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

Clearly for $n = 2$, the matrices in options (a), (b),

(c) do not tally with $\begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

152 (a)

We have,

$$A = [a_{ij}] \therefore |kA| = k^n |A|$$

153 (b)

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

154 (d)

$$\text{Given that, } A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + 0 & 0 + 0 \\ \alpha + 1 & 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$$

Also, $B = A^2$ (given)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$$

Clearly this is not satisfied by any real value of α

155 (b)

We have,

$$A(\text{adj } A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow |A| I = 4 I [\because A(\text{adj } A) = |A| I]$$

$$\Rightarrow |A| = 4$$

$$\Rightarrow |\text{adj } A| = |A|^2 [|\text{adj } A| = |A|^{n-1}]$$

156 (a)

Given, $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$

$$A^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^3 & 0 \\ 0 & \omega^3 \end{bmatrix}$$

Similarly, $A^{50} = \begin{bmatrix} \omega^{50} & 0 \\ 0 & \omega^{50} \end{bmatrix}$

$$= \begin{bmatrix} (\omega^2)^{16} \omega^2 & 0 \\ 0 & (\omega^2)^{16} \omega^2 \end{bmatrix}$$

$$= \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$$

$$= \omega^2 A$$

157 (a)

We have,

$$AB = I_3$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow x + y = 0$$

158 (a)

Let $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

$$\therefore \text{adj}(A) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

159 (a)

$$\because |A| = 0 - 1(1 - 9) + 2(1 - 6)$$

$$= 8 - 10 = -2 \neq 0$$

$$\text{adj}(A) = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-2} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

160 (a)

$$|f(\theta)| = 1(\cos^2 \theta + \sin^2 \theta) = 1$$

Now, $\text{adj}\{f(\theta)\} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore \{f(\theta)\}^{-1} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-\theta)$$

161 (a)

Given, $x + 4ay + az = 0$... (i)

$x + 3by + bz = 0$... (ii)

And $x + 2cy + cz = 0$... (iii)

For non-trivial solution

$$\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & 4a & a \\ 0 & 3b - 4a & b - a \\ 0 & 2c - 4a & c - a \end{vmatrix} = 0$$

$$\Rightarrow 1[(3b - 4a)(c - a) - 2(b - a)(c - 2a)] = 0$$

$$\Rightarrow 3bc - 3ab - 4ac + 4a^2$$

$$- 2(bc - 2ab - ac + 2a^2) = 0$$

$$\Rightarrow bc + ab - 2ac = 0$$

$$\Rightarrow ab + bc = 2ac$$

162 (d)

We know that

$$\text{rank}(AB) \leq \text{rank}(A)$$

$$\text{and, rank}(AB) \leq \text{rank}(B)$$

$$\therefore \text{rank}(AB) \leq \min(\text{rank } A, \text{rank } B)$$

163 (c)

Let $A = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ and $B = [b_{11} b_{12} b_{13} \cdots b_{1n}]$ be two

non-zero column and row matrices respectively

We have, $AB = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} & \cdots & a_{11}b_{1n} \\ a_{21}b_{11} & a_{21}b_{12} & a_{21}b_{13} & \cdots & a_{21}b_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1}b_{11} & a_{m1}b_{12} & a_{m1}b_{13} & \cdots & a_{m1}b_{1n} \end{bmatrix}$

Since A and B are non-zero matrices. Therefore, the matrix AB will also be a non-zero matrix. The matrix AB will have at least one non-zero element obtained by multiplying corresponding non-zero elements of A and B . All the two-rowed minors of A obviously vanish. But, A is a non-zero matrix.

Hence, $\text{rank}(A) = 1$

164 (c)

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$$

$$= - \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

165 (a)

If A is any square matrix, then

$$AA^{-1} = I \text{ and } A^{-1}I = A^{-1}$$

Since, $A^2 - A + I = O$

$$\Rightarrow A^{-1}A^2 - A^{-1}A + A^{-1}I = O$$

$$\Rightarrow (A^{-1}A)A - (A^{-1}A) + A^{-1} = O$$

$$\Rightarrow A - I + A^{-1} = O \Rightarrow A^{-1} = I - A$$

166 (a)

Since, B is invertible, therefore B^{-1} exists

$$\text{Now, rank}(A) = \text{rank}[(AB)B^{-1}] \leq \text{rank}(AB)$$

$$\text{But rank}(AB) \leq \text{rank}(A)$$

$$\therefore \text{rank}(AB) = \text{rank}(A)$$

167 (c)

Given, $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$ of order $n = 2$

$$\therefore |\text{adj}(A)| = |A|^{2-1} = \begin{vmatrix} 4 & 2 \\ 3 & 4 \end{vmatrix} = 10$$

168 (d)

$$\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & 0 \\ 0 & \cos^2 \theta + \sin^2 \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

169 (b)

Let A denote the matrix every element of which is unity. Then, all the 2-rowed minors of A obviously vanish. But A is a non-null matrix. Hence, rank of A is 1

170 (d)

As $\det(A) = \pm 1, A^{-1}$ exists
and $A^{-1} = \frac{1}{\det(A)} (\text{adj } A) = \pm (\text{adj } A)$
All entries in $\text{adj } (A)$ are integers.
 $\therefore A^{-1}$ has integer entries.

171 (c)

Since, A is invertible

$$\therefore |A| \neq 0 \Rightarrow \begin{vmatrix} 1 & 0 & -k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow 1(1-0) + k(0-k) \neq 0$$

$$\Rightarrow 1-k^2 \neq 0 \Rightarrow k \neq \pm 1$$

172 (b)

We have,

$$\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \frac{1}{1+\tan^2 \theta} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \frac{1}{1+\tan^2 \theta} \begin{bmatrix} 1-\tan^2 \theta & -2\tan \theta \\ 2\tan \theta & 1-\tan^2 \theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1-\tan^2 \theta}{1+\tan^2 \theta} & \frac{-2\tan \theta}{1+\tan^2 \theta} \\ \frac{2\tan \theta}{1+\tan^2 \theta} & \frac{1-\tan^2 \theta}{1+\tan^2 \theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow a = \cos 2\theta, b = \sin 2\theta$$

173 (c)

We have,

$$x^2 + y^2 + z^2 \neq 0$$

$$\Rightarrow \text{At least one of } x, y, z \text{ is non-zero}$$

Now,

$$x = cy + bz, y = az + cx, z = bx + ay$$

$$\Rightarrow x - cy - bz = 0$$

$$cx - y + az = 0$$

$$bx + zy - z = 0$$

As at least one of x, y, z is non-zero. Therefore, the above system of equations has non-trivial solutions

$$\therefore \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow a^2 + b^2 + c^2 + 2abc = 1$$

174 (c)

$$A^2 - 4A + 10I = A$$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} - 4 \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} + 10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -5 & -3-3k \\ 2+2k & -6+k^2 \end{bmatrix} - \begin{bmatrix} 4 & -12 \\ 8 & 4k \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 9-3k \\ -6+2k & 4+k^2-4k \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

$$\Rightarrow 9-3k = -3, -6+2k = 2 \quad \dots(i)$$

and $4+k^2-4k = k$

$$\Rightarrow k^2 - 5k + 4 = 0 \Rightarrow k = 4, 1$$

But $k = 1$ is not satisfied the Eq (i).

175 (a)

Given, $A^2 = 2A - I$

Now, $A^3 = A^2 \cdot A = 2A^2 = -IA$

$$= 2A^2 - A = 2(2A - I) - A$$

$$= 3A - 2I = 3A - (3-1)I$$

...

...

$$A^n = nA - (n-1)I$$

176 (c)

We have, $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$$\Rightarrow A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

177 (b)

It is given that A is an orthogonal matrix
 $\therefore A A^T = I = A^T A \Rightarrow A^{-1} = A^T$

178 (a)

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -2 & -3 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow -R_2$ and $R_2 \rightarrow R_2 - 2R_3$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 2R_2 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

179 (a)

Given that, $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$

$$\Rightarrow 2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

180 (b)

Since the given matrix is symmetric

$$\therefore (A)_{12} = (A)_{21} \Rightarrow x + 2 = 2x - 3 \Rightarrow x = 5$$

181 (d)

Given, $A = 3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$\therefore A^2 = 3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot 3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} 9A$$

$$\therefore A^4 = A^2 \cdot A^2 = 9A \cdot 9A = 81 \cdot 9A = 729A$$

182 (a)

Now, $\begin{vmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{vmatrix}$

$$= 1(\omega^3 - 1) - \omega^2(\omega^4 - \omega) + \omega(\omega^2 - \omega^2)$$

$$= 1(1 - 1) - \omega^2(\omega - \omega) + 0$$

$$= 0$$

Hence, matrix A is singular

183 (a)

Given system of equations are

$$x + y + z = 6, x + 2y + 3z = 10$$

$$\text{and } x + 2y + \lambda z = \mu$$

The given system of equations has infinite number of solutions, if any two equations will be same i.e., the last two equations will be same, if $\lambda = 3, \mu = 10$.

184 (a)

Given, $(A + B)(A - B) = A^2 - B^2$

$$\Rightarrow A^2 - AB + BA - B^2 = A^2 - B^2$$

$$\Rightarrow AB = BA$$

Now, $(ABA^{-1})^2 = (BAA^{-1})^2 = B^2$

185 (c)

Since diagonal elements of a skew-symmetric matrix are all zeros i.e. $a_{ii} = 0$ for all i

$$\therefore \text{tr}(A) = \sum_{i=1}^n a_{ii} = 0$$

186 (c)

$$\therefore P^6 = P(I - P) \quad \therefore P^2 = I - P$$

$$= PI - P^2 = PI - (I - P)$$

$$\text{Now, } P^4 = P \cdot P^3$$

$$\Rightarrow P^4 = P(2P - I)$$

$$\Rightarrow P^4 = 2P^2 - P$$

$$\Rightarrow P^4 = 2I - 2P - P$$

$$\Rightarrow P^4 = 2I - 3P$$

$$\text{And } P^5 = P(2I - 3P)$$

$$\Rightarrow P^5 = 2P - 3(I - P)$$

$$\Rightarrow P^5 = 5P - 3I$$

$$\text{Also, } P^6 = P(5P - 3I)$$

$$\Rightarrow P^6 = 5P^2 - 3P$$

$$\Rightarrow P^6 = 5(I - P) - 3P$$

$$\Rightarrow P^6 = 5I - 8P$$

$$\text{So, } n = 6$$

Alternate Solution

$$\therefore P^n = 5I - 8P$$

$$= 5(I - P) - 3P$$

$$= P(5P - 3I) \quad (\therefore P^2 = I - P)$$

$$= P(2P - 3P^2)$$

$$= P^2(2I - 3P)$$

$$= P^2[2(I - P) - P]$$

$$= P^2[2P^2 - P]$$

$$= P^3[2P - I]$$

$$= P^4[I - P]$$

$$= P^4 \cdot P^2 = P^6$$

$$\Rightarrow n = 6$$

187 (b)

$$A^2 = A \cdot A = AB \cdot A$$

$$= A \cdot BA = AB = A$$

189 (d)

Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$$

and $\text{adj } A = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{hence, } A^{-1} = \frac{1}{|A|} \text{adj } A = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{so, required element} = A_{13}^{-1} = 7$$

190 (a)

$$\therefore |A| = 1$$

and $A^c = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $\text{adj } A = (A^c)' = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-x)$$

191 (b)

$$\therefore |A| = 1(0 - 1) = -1$$

$\therefore$ Cofactors of A are

$$C_{11} = 0, C_{12} = 0, C_{13} = -1$$

$$C_{21} = 0, C_{22} = -1, C_{23} = 0$$

$$C_{31} = -1, C_{32} = 0, C_{33} = 0$$

$$\therefore A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = A$$

192 (b)

We have,

$$A^2 - 5I_2 - \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5A$$

$$\therefore k = 5$$

194 (b)

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\text{and } B = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

$$\therefore AX = B$$

$$\Rightarrow X = A^{-1}B$$

$$\text{Here, } A^{-1} = \frac{1}{6} \begin{bmatrix} 4 & 2 & 0 \\ -3 & 0 & 3 \\ 5 & -2 & -3 \end{bmatrix}$$

$$\therefore X = \frac{1}{6} \begin{bmatrix} 4 & 2 & 0 \\ -3 & 0 & 3 \\ 5 & -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 0 + 6 + 0 \\ 0 + 0 + 12 \\ 0 - 6 - 12 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

$$\text{Thus, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

195 (a)

The given system of equations can be rewritten as matrix form $AX = B$ as

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{Now, } |A| = 1(6 + 1) + 1(3 + 2) + 1(1 - 4) = 7 + 5 - 3 = 9 \neq 0$$

Since, $|A| \neq 0$. So, the given system of equations has only trivial solution. So, there is no non-trivial solution.

196 (d)

If matrix has no inverse it means the value of determinant should be zero.

$$\therefore \begin{vmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{vmatrix} = 0$$

If we put $x = 1$, then column Ist and IIIrd are

identical.

197 (b)

Since, $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix

$$\therefore \begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{vmatrix} = 0$$

$$\Rightarrow (2+x)(5-2) - 3(-5-2x) + 4(1+x) = 0$$

$$\Rightarrow 6 + 3x + 15 + 6x + 4 + 4x = 0$$

$$\Rightarrow 13x + 25 = 0 \Rightarrow x = -\frac{25}{13}$$

198 (a)

We have,

$$A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$$

$$\Rightarrow A \sim \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -4 & 2 \end{bmatrix} \quad \text{Applying } R_2 \rightarrow 2R_2 + R_3$$

$$\Rightarrow A \sim \begin{bmatrix} 2 & 3 & -5 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} \text{Applying } C_3 \rightarrow C_3 - 2C_2 \\ C_4 \rightarrow C_4 + C_2 \end{array}$$

$$\Rightarrow A \sim \begin{bmatrix} 2 & 3 & -5 & 7 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Applying } R_2 \leftrightarrow R_3$$

Clearly, $\begin{vmatrix} 2 & 3 \\ 0 & -2 \end{vmatrix} \neq 0$ and every minor of order 3 is zero

Hence, rank of A is 2

199 (b)

We have,

$$A^2 = AA = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$$

$$\Rightarrow \alpha = a^2 + b^2, \beta = 2ab$$

200 (d)

In a square matrix, the trace of A is defined as the sum of the diagonal elements

$$\text{Hence, trace of } A = \sum_{i=1}^n a_{ii}$$

201 (a)

Given system of equations is $x + 2y + 3z = 1$, $2x + y + 3z = 2$ and $5x + 5y + 9z = 5$

$$\text{Now, } \Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix}$$

$$= 1(9 - 15) - 2(18 - 15) + 3(10 - 5)$$

$$= -6 - 6 + 15$$

$$= 3 \neq 0$$

Hence, it has unique solution

202 (a)

$$\text{Let } \Delta = \begin{vmatrix} 4 & 2 & (1-x) \\ 5 & k & 1 \\ 6 & 3 & (1+x) \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$

$$\Rightarrow \Delta = \begin{vmatrix} 10 & 5 & 2 \\ 5 & k & 1 \\ 6 & 3 & 1+x \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - 2C_2$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 5 & 2 \\ 5-2k & k & 1 \\ 0 & 3 & 1+x \end{vmatrix}$$

$$\Rightarrow (5-2k)(5+5x-6) = 0$$

$$\Rightarrow k = \frac{5}{2}, \quad x = \frac{1}{5}$$

204 (b)

$$\text{Since, } \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \cdot \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

205 (c)

It is a direct consequence of the definition of rank

206 (a)

$$\begin{aligned} \text{Now, } A(x)A(y) &= (1-x)^{-1} \begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix} (1-y)^{-1} \begin{bmatrix} 1 & -y \\ -y & 1 \end{bmatrix} \\ &= [(1+xy) - (x+y)]^{-1} \begin{bmatrix} 1+xy & -(x+y) \\ -(x+y) & 1+xy \end{bmatrix} \\ &= \left(1 - \frac{x+y}{1+xy}\right)^{-1} \begin{bmatrix} 1 & -\frac{x+y}{1+xy} \\ -\frac{x+y}{1+xy} & 1 \end{bmatrix} \\ &= A(z) \end{aligned}$$

207 (a)

$$\begin{aligned} A^2(\alpha) &= \begin{vmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{vmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & 2\cos \alpha \sin \alpha \\ -2\sin \alpha \cos \alpha & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix} = A(2\alpha) \end{aligned}$$

208 (a)

Since, A is symmetric matrix, therefore $A^T = A$

Now, $(A^n)^T = (A^T)^n = A^n$

Hence, A^n is a symmetric matrix.

209 (a)

$$\begin{aligned} \text{Let } A &= \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \\ \therefore |A| &= \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} \\ &= 0 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \\ &= -(1-9) + 2(1-6) = 8-10 = -2 \end{aligned}$$

$$\text{and } \text{Adj } A = \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

$$\text{Hence, } A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$\begin{aligned} &= -\frac{1}{2} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix} \end{aligned}$$

210 (a)

We know that

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

Clearly, $\frac{1}{2}(A + A^T)$ is a symmetric matrix and

$\frac{1}{2}(A - A^T)$ is a skew-symmetric matrix

Now,

$$\begin{aligned} \frac{1}{2}(A + A^T) &= \frac{1}{2} \left\{ \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix} \right\} \\ \Rightarrow \frac{1}{2}(A + A^T) &= \frac{1}{2} \begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix} \end{aligned}$$

211 (c)

Since, the system of linear equations has a non-zero solution, then

$$\begin{bmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{bmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\begin{aligned} \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} &= 0 \\ \Rightarrow (3b-2a)(c-a) - (4c-2a)(b-a) &= 0 \\ \Rightarrow 3bc - 3ba - 2ac + 2a^2 &= 4bc - 2ab - 4ac + 2a^2 \\ &= 4bc - 2ab - 4ac + 2a^2 \end{aligned}$$

$$\Rightarrow 2ac = bc + ab$$

On dividing by abc both sides, we get

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\Rightarrow a, b, c \text{ are in HP.}$$

212 (c)

Given system of equations is

$$x - y + z = 3$$

$$2x + y - z = 2$$

$$\text{and } -3x - 2ky + 6z = 3$$

$\therefore$ The given system will have infinite solutions.

$$\therefore \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -3 & -2k & 6 \end{vmatrix} = 0$$

$$\Rightarrow 6k - 18 = 0 \Rightarrow k = 3$$

213 (a)

The product of two orthogonal matrix is an

orthogonal matrix

214 (b)

Given system of equations can be rewritten as
 $AX = B$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 5 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 14 \end{bmatrix}$$

$$\therefore |A| = 1(7 - 10) - 1(-7 - 6) + 1(5 + 3) \\ = -3 + 13 + 8 = 18 \neq 0$$

$\therefore$ Given system has unique solution.

215 (a)

Given, equations ($x + ay = 0, az + y = 0, ax + z = 0$) has infinite solutions.

$\therefore$ Using Cramer's rule, its determinant = 0

$$\Rightarrow \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1 + a^3 = 0 \Rightarrow a = -1$$

217 (a)

$$\text{Given that, } F(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow F(-\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore F(\alpha)F(-\alpha)$$

$$= \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha & 0 \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$\Rightarrow [F(\alpha)]^{-1} = F(-\alpha)$$

218 (b)

By using inverse of matrix, we know

$$|M^{-1}| = |M|^{-1} \text{ holds true}$$

$$(M^T)^{-1} = (M^{-1})^T \text{ holds true}$$

$$\text{and } (M^{-1})^{-1} = M \text{ holds true}$$

$$\text{but } (M^2)^{-1} = (M^{-1})^{-2} \text{ not true}$$

219 (a)

$$\text{Since, } A^2 - B^2 = (A - B)(A + B)$$

$$= A^2 - B^2 + AB - BA$$

$$\Rightarrow AB = BA$$

220 (c)

$$\text{Given, } x_1 + 2x_2 + 3x_3 = 0 \quad \dots(i)$$

$$2x_1 + 3x_2 + x_3 = 0 \quad \dots(ii)$$

$$3x_1 + x_2 + 2x_3 = 0 \quad \dots(iii)$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$$= 1(6 - 1) - 2(4 - 3) + 3(2 - 9)$$

$$= -18$$

Then, this system has the unique solution.

222 (d)

$$X^2 = X \cdot X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

For $n = 2$, no option is satisfied

Hence, option (d) is correct

223 (a)

We have,

$$F(\alpha)F(-\alpha)$$

$$= \begin{bmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow F(\alpha)F(-\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$\Rightarrow F(-\alpha) = [F(\alpha)]^{-1}$$

224 (b)

$$\text{We have, } (AA^T)^T = (A^T)^T A^T = AA^T$$

$$\therefore AA^T \text{ is symmetric matrix}$$

225 (c)

For any square matrix X , we have

$$X(\text{adj } X) = |X|I_n$$

Taking $X = \text{adj } A$, we have

$$(\text{adj } A)(\text{adj } (\text{adj } A)) = |\text{adj } A|I_n$$

$$\Rightarrow \text{adj } A(\text{adj } (\text{adj } A)) = |A|^{n-1}I_n [\because |\text{adj } A| \\ = |A|^{n-1}]$$

$$\Rightarrow (A \text{ adj } A)(\text{adj } (\text{adj } A)) = |A|^{n-1}A [\because A I_n = A]$$

$$\Rightarrow (|A|I_n)(\text{adj } (\text{adj } A)) = |A|^{n-1}A$$

$$\Rightarrow \text{adj } (\text{adj } A) = |A|^{n-2}A$$

226 (d)

Given equations are

$$(\alpha + 1)^3 x + (\alpha + 2)^3 y - (\alpha + 3)^3 = 0$$

$$(\alpha + 1)x + (\alpha + 2)y - (\alpha + 3) = 0$$

$$\text{and } x + y - 1 = 0$$

Since, this system of equations is consistent.

$$\therefore \begin{vmatrix} (\alpha + 1)^3 & (\alpha + 2)^3 & -(\alpha + 3)^3 \\ (\alpha + 1) & (\alpha + 2) & -(\alpha + 3) \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$$\text{Applying } C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 + C_1$$

$$\Rightarrow \begin{bmatrix} (\alpha + 1)^3 & (\alpha + 2)^3 - (\alpha + 1)^3 & -(\alpha + 1)^3 \\ (\alpha + 1) & (\alpha + 2) - (\alpha + 1) & -(\alpha + 1) \\ 1 & 1 & -1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} (\alpha + 1)^3 & 3\alpha^2 + 9\alpha + 7 & -6\alpha^2 - 24\alpha - 26 \\ (\alpha + 1) & 1 & -2 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= 0$$

$$\Rightarrow -2(3\alpha^2 + 9\alpha + 7) + 6\alpha^2 + 24\alpha + 26 = 0$$

$$\Rightarrow 6\alpha + 12 = 0 \Rightarrow \alpha = -2$$

227 (a)

We have,

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$

$$\Rightarrow \frac{1}{n}A^n = \begin{bmatrix} \frac{\cos n\theta}{n} & \frac{\sin n\theta}{n} \\ -\frac{\sin n\theta}{n} & \frac{\cos n\theta}{n} \end{bmatrix}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n}A^n = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

228 (c)

We have,

$$AB = O$$

$$\Rightarrow \begin{bmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha \end{bmatrix} \begin{bmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} \cos \alpha \cos \beta \cos(\alpha - \beta) & \cos \alpha \sin \beta \cos(\alpha - \beta) \\ \cos \beta \sin \alpha \cos(\alpha - \beta) & \sin \alpha \sin \beta \cos(\alpha - \beta) \end{bmatrix} = O$$

$$\Rightarrow \cos(\alpha - \beta) = 0$$

$$\Rightarrow \alpha - \beta \text{ is an odd multiple of } \frac{\pi}{2}$$

229 (c)

$$|A| |\text{adj } A| = |A|^n \text{ for order } n$$

$$\Rightarrow DD' = D^n$$

230 (c)

$$\text{Given, } \begin{bmatrix} 1+\omega & 2\omega \\ -2\omega & -b \end{bmatrix} + \begin{bmatrix} a & -\omega \\ 3\omega & 2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+\omega+a & \omega \\ \omega & 2-b \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix}$$

$$\Rightarrow 1+\omega+a=0, 2-b=1$$

$$\Rightarrow a=-1-\omega, b=1$$

$$\therefore a^2+b^2=(-1-\omega)^2+1^2$$

$$=1+\omega^2+2\omega+1^2$$

$$=0+\omega+1 (\because 1+\omega+\omega^2=0)$$

$$=1+\omega$$

232 (a)

$$\text{Given, } 2kx-2y+3z=0, x+ky+2z=0, 2x+kz=0$$

For non-trivial solution

$$\begin{vmatrix} 2k & -2 & 3 \\ 1 & k & 2 \\ 2 & 0 & k \end{vmatrix} = 0$$

$$\Rightarrow 2k(k^2-0)+2(k-4)+3(0-2k)=0$$

$$\Rightarrow 2k^3-4k-8=0$$

$$\Rightarrow (k-2)(2k^2+4k+4)=0$$

$$\Rightarrow k=2$$

233 (a)

$$AB = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2+0+15 & -2+2+0 \\ 4+0+0 & -4+2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}$$

234 (d)

$$A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$$

$$\therefore A^2 = B \Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \alpha^2 = 1 \text{ and } \alpha+1 = 5$$

Which is not possible at the same time.

$\therefore$ No real values of α exists.

235 (c)

We have,

$$E(\alpha)E(\beta)$$

$$= \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = E(\alpha+\beta)$$

Hence, option (c) is correct.

237 (c)

$$\text{We have, } A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} - \frac{1}{2} & \frac{1}{2} - \frac{1}{2} \\ -\frac{1}{2} + \frac{1}{2} & -\frac{1}{2} + \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$\therefore$ Matrix A is nilpotent

238 (d)

$$\text{Since, } A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\text{Now, } |A| = 1(0-2) + 1(2-3) + 2(4-0) = 5$$

$$\therefore A^{-1} = \frac{1}{5} \begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$$

$$\text{Now, } A^{-1}B = \frac{1}{5} \begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

239 (a)

Since, A is a skew-symmetric matrix. Therefore,

$$A^T = -A \Rightarrow |A^T| = |-A|$$

$$\Rightarrow |A| = (-1)^n |A|$$

Also, n is odd

$$\therefore 2|A| = 0 \Rightarrow |A| = 0$$

$$\text{Thus, } |\text{adj } A| = |A|^2 = 0$$

240 (d)

Given System of equations are

$$x+3y+2z=0$$

$$3x + y + z = 0$$

$$\text{and } 2x - 2y - z = 0$$

$$\text{Now, } \Delta = \begin{vmatrix} 1 & 3 & 2 \\ 3 & 1 & 1 \\ 2 & -2 & -1 \end{vmatrix}$$

$$= 1(-1 + 2) - 3(-3 - 2) + 2(-6 - 2)$$

$$= 1 + 15 - 16$$

$$= 0$$

Since, determinant is zero, then it has infinitely many solutions.

242 (b)

$$\text{Let } \Delta = \begin{vmatrix} a_1 & a_2 \\ a_4 & a_5 \end{vmatrix}$$

$$= a_1 a_5 - a_2 a_4$$

$$= a_1(a_1 + 4d) - (a_1 + d)(a_1 + 3d)$$

$$= a_1^2 + 4a_1 d - a_1^2 - 4a_1 d - 3d^2 = -3d^2 \neq 0$$

Hence, given system of equations has unique solution.

243 (b)

$$\therefore I_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}, |I_n| = 1$$

$$\text{adj}(I_n) = I_n$$

$$\therefore (I_n)^{-1} = I_n$$

244 (c)

We know that

$$(\text{adj } A)^T = \text{adj } A^T$$

$$\Rightarrow \text{adj } A^T - (\text{adj } A)^T = O \text{ (Null matrix)}$$

245 (c)

$$\text{Given, } \begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 10 \end{bmatrix}$$

It is of the form $AX = B$... (i)

$$|A| = 2(3 + 2) + 1(1 + 3) + 3(2 - 9) = -7$$

$$\therefore \text{adj}(A) = \begin{bmatrix} 5 & 7 & -8 \\ -4 & -7 & 5 \\ -7 & -7 & 7 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{-7} \begin{bmatrix} 5 & 7 & -8 \\ -4 & -7 & 5 \\ -7 & -7 & 7 \end{bmatrix}$$

$$\text{From Eq.(i), } X = -\frac{1}{7} \begin{bmatrix} 5 & 7 & -8 \\ -4 & -7 & 5 \\ -7 & -7 & 7 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{7} \begin{bmatrix} -7 \\ -14 \\ -21 \end{bmatrix}$$

$$\Rightarrow x = 1, y = 2, z = 3$$

247 (b)

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 4 & 1 & 1 \end{bmatrix}$$

$$= 1(1 + 1) + 1(2 - 4) + 1(2 - 4) = 6 \neq 0$$

Hence, it has unique solution.

248 (d)

$$\text{Given, } A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\text{Now, } |A| = \cos^2 \theta + \sin^2 \theta = 1 \neq 0.$$

$\therefore A$ is invertible.

249 (b)

$$|A| = -1 \text{ and } \text{adj } A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A$$

250 (b)

$$A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & a+6 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{bmatrix}$$

[using $R_1 \rightarrow R_1 + R_3$ and $R_2 \rightarrow R_2 - 2R_3$]

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{bmatrix} \text{ [using } R_1 \rightarrow R_1 + R_2]$$

$$\text{When } a = -6, A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -2 & -5 \end{bmatrix} \therefore \rho(A) = 1$$

$$\text{When } a = 6, A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -12 \\ 1 & -2 & 7 \end{bmatrix}, \therefore \rho(A) = 2$$

$$\text{When } a = 1, A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -7 \\ 1 & -2 & 2 \end{bmatrix}, \therefore \rho(A) = 2$$

$$\text{When } a = 2, A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -8 \\ 1 & -2 & 3 \end{bmatrix} \therefore \rho(A) = 2$$

251 (b)

$$\text{Let } D = \begin{bmatrix} d_1 & 0 & 0 & \dots & 0 \\ 0 & d_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & d_n \end{bmatrix}$$

$$\text{Then, } |D| = d_1 d_2 \dots d_n$$

$$\text{Now, Cofactor of } D_{11} = d_2 d_3 \dots d_n$$

$$\text{Cofactor of } D_{22} = d_1 d_3 \dots d_n \text{ etc}$$

$$\text{And, Cofactor of } D_{ij} = 0 \text{ when } i \neq j$$

$$\therefore D^{-1} = \frac{1}{|D|} \text{adj } D$$

$$= \frac{1}{d_1 d_2 \dots d_n} \begin{bmatrix} d_2 d_3 \dots d_n & 0 & 0 & \dots & 0 \\ 0 & d_2 d_3 \dots d_n & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & d_1 d_2 \dots d_n \end{bmatrix}$$

$$\therefore D^{-1} = \begin{bmatrix} \frac{1}{d_1} & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{d_2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \frac{1}{d_n} \end{bmatrix} = \text{diag}(s_1^{-1} d_2^{-1} \dots d_n^{-1})$$

252 (d)

$$\therefore A^2 = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$$

$$\therefore f(A) = A^2 + 4A - 5$$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$$

253 (a)

$$A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$\text{Again now, } 4A - 3I = 4 \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$\therefore A^2 = 4A - 3I$$

254 (d)

$$\therefore (AB)^{-1} = B^{-1}A^{-1}$$

255 (d)

$$|A| = -8$$

$$\text{adj}(A) = \begin{bmatrix} 0 & 0 & -4 \\ 0 & -4 & 0 \\ -4 & 0 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-8} \begin{bmatrix} 0 & 0 & -4 \\ 0 & -4 & 0 \\ -4 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1/2 \\ 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \end{bmatrix}$$

256 (b)

Since given system of equations possesses a non-zero solution.

$$\therefore \Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & -a & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow a(-a-1) - 1(1-1) + 1(1+a) = 0$$

$$\Rightarrow a^2 = 1 \Rightarrow a = \pm 1$$

257 (a)

$$\text{Now, } (A + A^T)^T = A^T + (A^T)^T = A^T + A$$

$\therefore A + A^T$ is symmetric matrix.

258 (c)

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} = 14$$

$$\therefore (\text{adj}(\text{adj } A)) = |A|^{n-2}A = 14^{3-2}A = 14A$$

$$\therefore |\text{adj}(\text{adj } A)| = |14A| = 14^3|A| = 14^4$$

259 (b)

$$\text{Given, } \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} x+y+z \\ x-2y-2z \\ x+3y+z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

On Comparing both sides, we get

$$x + y + z = 0 \quad \dots(i)$$

$$x - 2y - 2z = 3 \quad \dots(ii)$$

$$\text{and } x + 3y + z = 4 \quad \dots(iii)$$

On solving Eqs. (i), (ii) and (iii), we get

$$x = 1, y = 2 \text{ and } z = -3$$

260 (a)

$$|A| = \begin{vmatrix} 1 & 2 \\ -4 & -1 \end{vmatrix}$$

$$= -1 + 8 = 7$$

$$\text{adj}A = \begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{7} \begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$$

261 (c)

It is given that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & a+b \\ c & c+d \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix}$$

$$\Rightarrow a+c = a, a+b = b+d, c+d = d$$

$$\Rightarrow c = 0 \text{ and } a = d$$

262 (a)

$$AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$$

263 (a)

Let A be a skew-symmetric matrix of odd order $(2n+1)$ say. Since A is skew-symmetric

$$\therefore A^T = -A$$

$$\Rightarrow |A^T| = |-A|$$

$$\Rightarrow |A^T| = (-1)^{2n+1}|A|$$

$$\Rightarrow |A^T| = -|A|$$

$$\Rightarrow |A| = -|A| \Rightarrow 2|A| = 0 \Rightarrow |A| = 0$$

264 (a)

$$\text{As, } PP^T = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow PP^T = I \text{ or } P^T = P^{-1} \dots(i)$$

$$\text{As, } Q = PAP^T$$

$$\therefore P^T Q^{2005} P = P^T [PAP^T] (PAP^T)^T \dots 2005 \text{ times} P$$

$$= \frac{(P^T P) A (P^T P) A (P^T P) \dots (P^T P) A (P^T P)}{2005 \text{ times}}$$

$$= IA^{2005} = A^{2005}$$

$$\therefore A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \dots \text{and so on}$$

$$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow P^T Q^{2005} P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

265 (c)

$$\text{Given, } 2X + 3Y = 0 \quad \dots (i)$$

$$\text{and } X + 2Y = I \quad \dots(ii)$$

$$\text{where } O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On solving Eqs. (i) and (ii), we get

$$X = -3I = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

266 (d)

Subtracting the addition of first two equations

from third equation, we get
 $0 = -5$ which is an absurd result.

267 (d)

$$\text{Given } A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} x & -2 \\ 3 & 7 \end{vmatrix} = 7x + 6$$

$$\therefore A^{-1} = \frac{1}{7x+6} \begin{bmatrix} 7 & 2 \\ -3 & x \end{bmatrix}$$

$$\text{But given } A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ -\frac{3}{34} & \frac{2}{17} \end{bmatrix}$$

$$\therefore \frac{7}{7x+6} = \frac{7}{34}$$

$$\Rightarrow 7x+6 = 34 \Rightarrow 7x = 28 \Rightarrow x = 4$$

268 (d)

(a) It is clear that A is not a zero matrix.

$$(b) (-1)I = -1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$$

ie, $(-1)I \neq A$

$$(c) |A| = 0 \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} - 0 \begin{vmatrix} 0 & 0 \\ -1 & 0 \end{vmatrix} - 1 \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} \\ = 0 - 0 - 1(-1) = 1$$

Since, $|A| \neq 0$ so A^{-1} exists.

$$(d) A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^2 = I$$

270 (a)

$$\text{Since, } A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \\ = \begin{bmatrix} 1+4+0 & 0+2-1 & 0+0-3 \\ 3+0+0 & 0+0+2 & 0+0+6 \\ 4+10+0 & 0+5+0 & 0+0+0 \end{bmatrix} \\ = \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$$

271 (c)

We have,

$$[F(x) G(y)]^{-1} = [G(y)]^{-1} [f(x)]^{-1} \\ \Rightarrow [F(x) G(y)]^{-1} = G(-y) F(-x)$$

272 (a)

$$A^{-1} = \frac{1}{1+10} \begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$$

Also, $A^{-1} = xA + yI$

$$\Rightarrow \frac{1}{11} \begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} x & 2x \\ -5x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$$

$$\Rightarrow x + y = \frac{1}{11}, 2x = \frac{-2}{11}$$

$$\Rightarrow x = \frac{-1}{11}, y = \frac{2}{11}$$

273 (d)

$$\text{Now, } (AB)^T = B^T A^T$$

274 (c)

On comparing corresponding elements, we get

$$x + y + z = 9$$

$$x + y = 5$$

$$\text{and } y + z = 7$$

On solving these, we get $x = 2, y = 3, z = 4$

$$\Rightarrow (x, y, z) = (2, 3, 4)$$

275 (c)

$$A.A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \\ = \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix} \\ \Rightarrow A^2 = 0$$

$\therefore A$ is nilpotent matrix of order 2.

276 (d)

Since A, B and C are non-singular matrices, then

$$(AB^{-1}C)^{-1} = C^{-1}(AB^{-1})^{-1}$$

$$= C^{-1}((B^{-1})^{-1}A^{-1}) = C^{-1}BA^{-1}$$

277 (a)

Given matrix is invertible

$$\Rightarrow \begin{vmatrix} \lambda - 1 & 4 \\ -3 & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow \lambda(0 - 1) + 1(-6 + 1) + 4(-3 - 0) \neq 0$$

$$\Rightarrow -\lambda - 5 - 12 \neq 0$$

$$\Rightarrow \lambda \neq -17$$

278 (d)

From Eqs. (ii) and (iii), we get

$$\frac{3y^2}{b^2} - \frac{3z^2}{c^2} = 0$$

$$\Rightarrow \frac{z^2}{c^2} = \frac{y^2}{b^2}$$

On putting this value in Eq. (i), we get

$$\frac{2x^2}{a^2} - \frac{2y^2}{b^2} = 0$$

$$\Rightarrow \frac{x^2}{a^2} = \frac{y^2}{b^2}$$

$$\therefore \frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{z^2}{c^2} = k^2 \text{ (say)}$$

$$\Rightarrow x = \pm ka, y = \pm kb, z = \pm kc, \forall k \in R$$

279 (b)

$$\text{We have, } A = \begin{bmatrix} 1 & \log_b a \\ \log_a b & 1 \end{bmatrix}$$

$$\therefore |A| = 1 - \log_a b \log_b a = 1 - 1 = 0$$

280 (a)

$$|A| = 4 - 6 = -2$$

$$\text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = -\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

281 (b)

$$\text{Since, } P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix} \text{ and } Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$$

$$\begin{aligned} \therefore PQ &= \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix} \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix} \\ &= \begin{bmatrix} -i^2 - i^2 & i^2 + i^2 \\ i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix} \\ &= \begin{bmatrix} 1+1 & -1-1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} \end{aligned}$$

282 (d)

$$B = \text{adj}(A) = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$$

$$\text{Therefore, } \text{adj}(B) = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix}$$

$$\text{Now, } |\text{adj } B| = \begin{vmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{vmatrix} = 625$$

$$\text{and } |C| = 125|A| = 125 \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 5 & 0 \end{vmatrix} = 625$$

$$\therefore \frac{|\text{adj}(B)|}{|C|} = \frac{625}{625} = 1$$

Alternate

$$|A| = 1(0+3) + 1(0+6) + (0-4)$$

$$\text{Now, } \text{adj } B = \text{adj}(\text{adj } A)$$

$$= |A|A = 5A$$

$$\therefore \frac{|\text{adj } B|}{|C|} = \frac{|5A|}{|5A|} = 1$$

283 (d)

$$\text{Given, } A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} = A^T$$

284 (b)

The given system of equations posses non-zero solutions,

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a \\ 1 & -a & 1 \end{bmatrix} = 0$$

$$\text{Applying } R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & a-1 \\ 0 & -a-1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow 1(0 - (a^2 - 1)) = 0$$

$$\Rightarrow a^2 = 1 \Rightarrow a = \pm 1$$

285 (a)

$$\text{Given, } x \begin{bmatrix} -3 \\ 4 \end{bmatrix} + y \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \end{bmatrix}$$

$$\therefore -3x + 4y = 10 \quad \dots(i)$$

$$\text{and } 4x + 3y = -5 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x = -2, y = 1$$

287 (a)

$$|A| = 5 + 6 = 11$$

$$\text{and } \text{adj } A = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{11} \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$$

288 (c)

We know that, if

$$A^n = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} = \text{diag } [d_1 d_2 d_3]$$

Then,

$$A^n = \begin{bmatrix} d_1^n & 0 & 0 \\ 0 & d_2^n & 0 \\ 0 & 0 & d_3^n \end{bmatrix} = \text{diag } [d_1^n d_2^n d_3^n]$$

$$\therefore A^5 = \begin{bmatrix} 2^5 & 0 & 0 \\ 0 & 2^5 & 0 \\ 0 & 0 & 2^5 \end{bmatrix} = 16 A$$

289 (c)

$$AB = I \Rightarrow B = A^{-1}$$

$$= \frac{1}{1 + \tan^2 \theta} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2 \theta} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}$$

$$\Rightarrow (\sec^2 \theta) B = \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} = A(-\theta)$$

290 (b)

It is given that $A = [a_{ij}]$ is a skew-symmetric matrix

$$a_{ij} = -a_{ji} \text{ for all } i, j$$

$$\Rightarrow a_{ii} = -a_{ii} \text{ for all } i$$

$$\Rightarrow 2 a_{ii} = 0 \text{ for all } i \Rightarrow a_{ii} = 0 \text{ for all } i$$

291 (c)

We know that, if $A = \text{diag. } (d_1 d_2, \dots, d_n)$ is a diagonal matrix, then for any $k \in N$

$$A^k = \text{diag } (d_1^k, d_2^k, \dots, d_n^k)$$

$$\text{Here, } A = \text{diag. } (a, a, a)$$

$$\therefore A^n = \text{diag } (a^n, a^n, a^n) = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$$

292 (b)

We have,

$$(A B - B A)^T = (A B)^T - (B A)^T = B^T A^T - A^T B^T$$

$$\Rightarrow (A B - B A)^T = B A - A B [\because A^T = A, B^T = B]$$

$$\Rightarrow (A B - B A)^T = -(A B - B A)$$

So, $A B - B A$ is skew-symmetric matrix

293 (b)

Since $A B$ exists

$$\therefore \text{No. of rows in } B = \text{No. of columns in } A$$

$\Rightarrow$ No. of rows in $B = n$

Also, $B A$ exists

$\Rightarrow$ No. of columns in $B =$ No. of rows in A

$\Rightarrow$ No. of columns in $= m$

Hence, B is of order $n \times m$

294 (c)

We have,

$$(k A)(\text{adj } k A) = |k A| I_n$$

$$\Rightarrow k(A \text{ adj } k A) = k^n |A| I_n [\because |k A| = k^n |A|]$$

$$\Rightarrow A(\text{adj } k A) = k^{n-1} |A| I_n$$

$$\Rightarrow A \text{ adj } (k A) = k^{n-1} A(\text{adj } A) [\because A \text{ adj } A = |A| I_n]$$

$$\Rightarrow A \text{ adj } (k A) = A(k^{n-1} \text{adj } A)$$

$$\Rightarrow A^{-1}(A \text{ adj } (k A)) = A^{-1}(A(k^{n-1} \text{adj } A))$$

$$\Rightarrow (A^{-1} A)(\text{adj } k A) = (A^{-1} A)(k^{n-1} \text{adj } A)$$

$$\Rightarrow I(\text{adj } k A) = I(k^{n-1} \text{adj } A)$$

$$\Rightarrow \text{adj } k A = k^{n-1}(\text{adj } A)$$

295 (c)

It has a non-zero solution if

$$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow -6k + 6 = 0$$

$$\Rightarrow k = 1$$

296 (b)

$$(aI + bA)^2 = (aI + bA)(aI + bA)$$

$$= a^2 I^2 + aI(bA) + bA(aI) + (bA)^2$$

$$\text{Now, } I^2 = I \text{ and } IA = A$$

$$\therefore (aI + bA)^2 = a^2 I + 2abA + b^2(A^2)$$

$$\text{Now, } A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$$\therefore (aI + bA)^2 = a^2 I + 2abA$$

297 (b)

Since A is orthogonal matrix

$$\therefore A A^T = I = A^T A$$

$$\Rightarrow |A A^T| = |I| = |A^T A|$$

$$\Rightarrow |A| |A^T| = 1 = |A^T| |A|$$

$$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$$

298 (b)

Since, given system of equations has no solution, $\Delta = 0$ and any one amongst $\Delta x, \Delta y, \Delta z$ is non-zero.

$$\text{Where } \Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$\text{And } \Delta z = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -4 \\ 1 & 1 & \lambda \end{vmatrix} = 6 \neq 0$$

$$\Rightarrow \lambda = 1$$

299 (c)

Since, A is an idempotent matrix, therefore $A^2 = A$

$$\Rightarrow \begin{bmatrix} 2 & -2 & -16-4x \\ -1 & 3 & 16+4x \\ 4+x & -8-2x & -12+x^2 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$$

On comparing, $16 + 4x = 4$

$$\Rightarrow x = -3$$

300 (a)

We have,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } \text{adj } A = \begin{bmatrix} 6 & -2 & -6 \\ -4 & 2 & x \\ y & -1 & 1 \end{bmatrix}$$

$$\text{Clearly, } |A| = 6 - 8 + 4 = 2$$

$$\therefore A(\text{adj } A) = |A| I$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 6 & -2 & -6 \\ -4 & 2 & x \\ y & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2y-2 & 0 & 2x-18 \\ 0 & 2 & 3x-12 \\ 2y-4 & 0 & x-2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow 2y - 2 = 2, 2y - 4 = 0, 2x - 8 = 0, 3x - 12 = 0, x - 2 = 2$$

$$\Rightarrow x = 4, y = 2 \Rightarrow x + y = 6$$

301 (a)

$$\text{Since, } \begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$$

$$\Rightarrow x + y = 4 \quad \dots(i)$$

$$x - y = 0 \quad \dots(ii)$$

$$2x + z = 7 \quad \dots(iii)$$

$$\text{and } 2z + w = 10 \quad \dots(iv)$$

On solving these equations, we get

$$x = 2, y = 2, z = 3, w = 4$$

302 (b)

We have,

$$A = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow A^2 = -I \Rightarrow A^2 + I = O$$

303 (d)

$$m[-3 \ 4] + n[4 \ -3] = [10 \ -11]$$

$$\Rightarrow [-3m + 4n \ 4m - 3n] = [10 \ -11]$$

$$\Rightarrow -3m + 4n = 10 \quad \dots(i)$$

$$\text{and } 4m - 3n = -11 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$n = 1, m = -2$$

$$\text{Now, } 3m + 7n = 3(-2) + 7(1) = 1$$

304 (b)

We know that

$$A(\text{adj } A) = |A| I$$

$$\text{If } A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}, \text{ then } |A| = 1$$

$$\therefore A(\text{adj } A) = I \Rightarrow KI = I \Rightarrow k = 1$$

305 (a)

$$\because |A| = 3, \text{adj}(A) = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow (A^{-1})^3 = \frac{1}{27} \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}^3$$

$$= \frac{1}{27} \begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$$

306 (d)

$$\text{Given, } M = [a_{uv}]_{n \times n}$$

$$= [\sin(\theta_u - \theta_v) + i \cos(\theta_u - \theta_v)]$$

$$\Rightarrow \bar{M} = [\sin(\theta_v - \theta_u) - i \cos(\theta_u - \theta_v)]$$

$$\Rightarrow (\bar{M})^T = [\sin(\theta_v - \theta_u) - i \cos(\theta_v - \theta_u)]$$

$$= [-\sin(\theta_u - \theta_v) - i \cos(\theta_u - \theta_v)]$$

$$= -[\sin(\theta_u - \theta_v) + i \cos(\theta_u - \theta_v)]$$

$$= -M$$

307 (a)

If given system of equations have infinitely many solutions, then

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ \lambda & -1 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 2(-4 + 1) + 1(2 - \lambda) + 1(-1 + 2\lambda) = 0$$

$$\Rightarrow -6 + 2 - \lambda - 1 + 2\lambda = 0$$

$$\Rightarrow \lambda - 5 = 0$$

$$\Rightarrow \lambda = 5$$

308 (c)

If $AB = O$, then A and B may be equal to O individually. It is not necessary in any condition

309 (b)

$$E(\theta)E(\phi) = \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix}$$

$$\times \begin{bmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta \cos^2 \phi + \cos \theta \sin \theta \cos \phi \sin \phi \\ \cos \theta \sin \theta \cos^2 \phi + \sin^2 \theta \cos \phi \sin \phi \end{bmatrix}$$

$$\begin{bmatrix} \cos^2 \theta \cos \phi \sin \phi + \cos \theta \sin \theta \sin^2 \phi \\ \cos \theta \sin \theta \cos \phi \sin \phi + \sin^2 \theta \sin^2 \phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta \cos \phi \cos(\theta - \phi) \cos \theta \sin \phi \cos(\theta - \phi) \\ \cos \phi \sin \theta \cos(\theta - \phi) \sin \theta \sin \phi \cos(\theta - \phi) \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} \cos \theta \cos \phi \cos(2n+1)\frac{\pi}{2} \\ \cos \theta \sin \phi \cos(2n+1)\frac{\pi}{2} \\ \cos \theta \sin \phi \cos(2n+1)\frac{\pi}{2} \\ \sin \theta \sin \phi \cos(2n+1)\frac{\pi}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \left[\because \cos(2n+1)\frac{\pi}{2} = 0 \right]$$

310 (c)

$$\text{Given, } x \sin 3\theta - y + z = 0$$

$$x \cos 2\theta + 4y + 3z = 0 \text{ and } 2x + 7y + 7z = 0$$

For non-trivial solution.

$$\begin{vmatrix} \sin 3\theta - 1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

$$\Rightarrow \sin 3\theta(28 - 21)$$

$$- \cos 2\theta(-7 - 7) + 2(-3 - 4)$$

$$= 0$$

$$\Rightarrow 7 \sin 3\theta + 14 \cos 2\theta - 14 = 0$$

$$\Rightarrow 7(3 \sin \theta - 4 \sin^3 \theta) + 14(1 - 2 \sin^2 \theta) - 14 = 0$$

$$\Rightarrow -28 \sin^3 \theta - 28 \sin^2 \theta + 21 \sin \theta = 0$$

$$\Rightarrow -7 \sin \theta(4 \sin^2 \theta + 4 \sin \theta - 3) = 0$$

$$\Rightarrow \sin \theta(2 \sin \theta + 3)(2 \sin \theta - 1) = 0$$

$$\Rightarrow \sin \theta = 0, \sin \theta = \frac{1}{2} \left(\because \sin \theta \neq -\frac{3}{2} \right)$$

$$\Rightarrow \theta = n\pi, n\pi + (-1)^n \frac{\pi}{6}$$

311 (c)

$$\because kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$\Rightarrow k \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$\Rightarrow 2k = 3a, 3k = 2b, -4k = 24$$

$$\Rightarrow a = \frac{2k}{3}, b = \frac{3k}{2}, k = -6$$

$$\therefore a = -4, b = -9, k = -6$$

312 (d)

$$\because \text{adj}(\text{adj } A) = |A|^{n-2} A$$

$$\text{Here } n = 3$$

$$\Rightarrow \text{adj}(\text{adj } A) = |A|A$$

313 (c)

$$A^5 = A^2 A^2 A = (A + I)(A + I)A$$

$$= (A^2 + 2AI + I^2)A$$

$$= (A + I + 2A + I)A = (3A + 2I)A$$

$$= 3A^2 + 2IA = 3(A + I) + 2IA$$

$$= 3A + 3I + 2A = 5A + 3I$$

314 (c)

Matrices $A + B$ and AB are defined only if both A and B are of same order $n \times n$.

315 (a)

$$UV + XY = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

$$= 6 - 6 + 4 + [0 + 4 + 12] = [4] + [16] = [20]$$

316 (b)

$$\text{For matrix } A = \begin{bmatrix} a & 2 \\ 2 & 4 \end{bmatrix} \text{ to be singular,}$$

$$\begin{vmatrix} a & 2 \\ 2 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 4a - 4 = 0$$

$$\Rightarrow a = 1$$

317 (a)

$$\text{adj}(A) = \begin{bmatrix} 4y & -x \\ -x^2 & 1 \end{bmatrix}$$

$$\therefore \text{adj}(A) + B = \begin{bmatrix} 4y & -x \\ -x^2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4y-3 & -x+1 \\ -x^2+1 & 1+0 \end{bmatrix}$$

$$\Rightarrow 4y-3=1 \Rightarrow y=1$$

$$\text{and } -x+1=0 \Rightarrow x=1$$

318 (c)

$$|A| = 1. (\cos^2 \alpha + \sin^2 \alpha) = 1$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \text{adj}(A) = \text{adj}(A)$$

319 (a)

We have,

$$(A+B)(A-B) = A^2 - AB + BA - B^2$$

So, option (a) is correct.

320 (a)

$$\text{Given, } (1-x)f(x) = 1+x$$

$$\Rightarrow (I-A)f(A) = (I+A) \quad (\because \text{Put } x=A)$$

$$\Rightarrow f(A) = (I-A)^{-1}(I+A)$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \right)$$

$$\Rightarrow f(A) = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}}{-4}$$

$$= \frac{\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}}{-4} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

321 (b)

We have,

$$AA^{-1} = I$$

$$\Rightarrow \det(AA^{-1}) = \det(I)$$

$$\Rightarrow \det(A) \det(A^{-1})$$

$$= 1 \left[\because \det(AB) = \det(A) \det(B) \right. \\ \left. \text{and } \det(I) = 1 \right]$$

$$\Rightarrow \det(A^{-1}) = \frac{1}{\det(A)}$$

322 (b)

$$\text{Since } AB = 3I$$

$$\Rightarrow A^{-1}AB = 3IA^{-1}$$

$$\Rightarrow B = 3A^{-1}$$

$$\Rightarrow A^{-1} = \frac{B}{3}$$

323 (b)

$$\text{We have, } \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2-x_1 & y_2-y_1 & 0 \\ x_3-x_1 & y_3-y_1 & 0 \end{vmatrix} = 0$$

[using $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$]

$\therefore$ The given points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear, therefore the rank of matrix is always greater than 0 and less than 3.

324 (a)

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$\text{And } 4A - 3I = 4 \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$\therefore A^2 = 4A - 3I$$

325 (a)

$$\text{Given, } A = \begin{bmatrix} -2 & 6 \\ -5 & 7 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 7 & -6 \\ 5 & -2 \end{bmatrix}$$

326 (c)

$$\text{Given, } A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$$

$$\Rightarrow |A| = 1$$

$$\therefore A \text{ adj}(A) = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |A|I$$

327 (c)

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

$$\text{And } BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

If $AB = BA$, then $a = b$

Hence, $AB = BA$ is possible for infinitely many values of B 's.

328 (b)

We have,

$$2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix} \dots (i)$$

$$\text{and } A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix} \dots (ii)$$

On multiplying Eq.(ii) by 2 and then subtracting Eq.(i) from Eq.(ii), we get

$$B = 2 \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$$

329 (b)

Determinant of unit matrix of any order is 1

330 (b)

$$AB = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} 3 & 17 \\ 2 & -20 \end{bmatrix}$$

$$\Rightarrow |AB| = 102 - 20$$

$$= 100$$

331 (a)

$$\text{We have, } A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} \text{ and } A' = \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}$$

$$\therefore \text{Symmetric matrix, } B = \frac{A+A'}{2}$$

$$\begin{aligned}
&= \frac{1}{2} \left\{ \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 11 \end{bmatrix} \right\} \\
&= \frac{1}{2} \begin{bmatrix} 12 & 12 & 14 \\ 12 & 4 & 10 \\ 14 & 10 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}
\end{aligned}$$

332 (b)

Since A is non-singular. Therefore, A^{-1} exists

Now, $A(\text{adj } A) = |A|I = (\text{adj } A)A$

$$\Rightarrow |A||\text{adj } A| = |A|^n = |\text{adj } A||A|$$

$$\Rightarrow |\text{adj } A| = |A|^{n-1} [\because |A| \neq 0]$$

333 (a)

$$\because A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix} = \begin{bmatrix} 2 & -1+a \\ 6 & -1+b \end{bmatrix}$$

$$\Rightarrow (A + B)^2 = \begin{bmatrix} 2 & -1+a \\ 6 & -1+b \end{bmatrix} \begin{bmatrix} 2 & -1+a \\ 6 & -1+b \end{bmatrix}$$

$$\begin{bmatrix} -2+6a & -1+a-b+ab \\ 6+6b & -5+6a-2b+b^2 \end{bmatrix}$$

$$\text{and } A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{aligned}
\text{Also, } B^2 &= \begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix} \begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix} \\
&= \begin{bmatrix} 1+4a & a+ab \\ 4+4b & 4a+b^2 \end{bmatrix}
\end{aligned}$$

$$\text{Given, } (A + B)^2 = A^2 + B^2$$

$$\begin{aligned}
\therefore \begin{bmatrix} -2+6a & -1+a-b+ab \\ 6+6b & -5+6a-2b+b^2 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1+4a & a+ab \\ 4+4b & 4a+b^2 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} -2+6a & -1+a-b+ab \\ 6+6b & -5+6a-2b+b^2 \end{bmatrix} \\
&= \begin{bmatrix} 4a & a+ab \\ 4+4b & -1+4a+b^2 \end{bmatrix}
\end{aligned}$$

On comparing both sides, we get

$$-2+6a = 4a \quad \text{and} \quad 6+6b = -1+4a+b^2$$

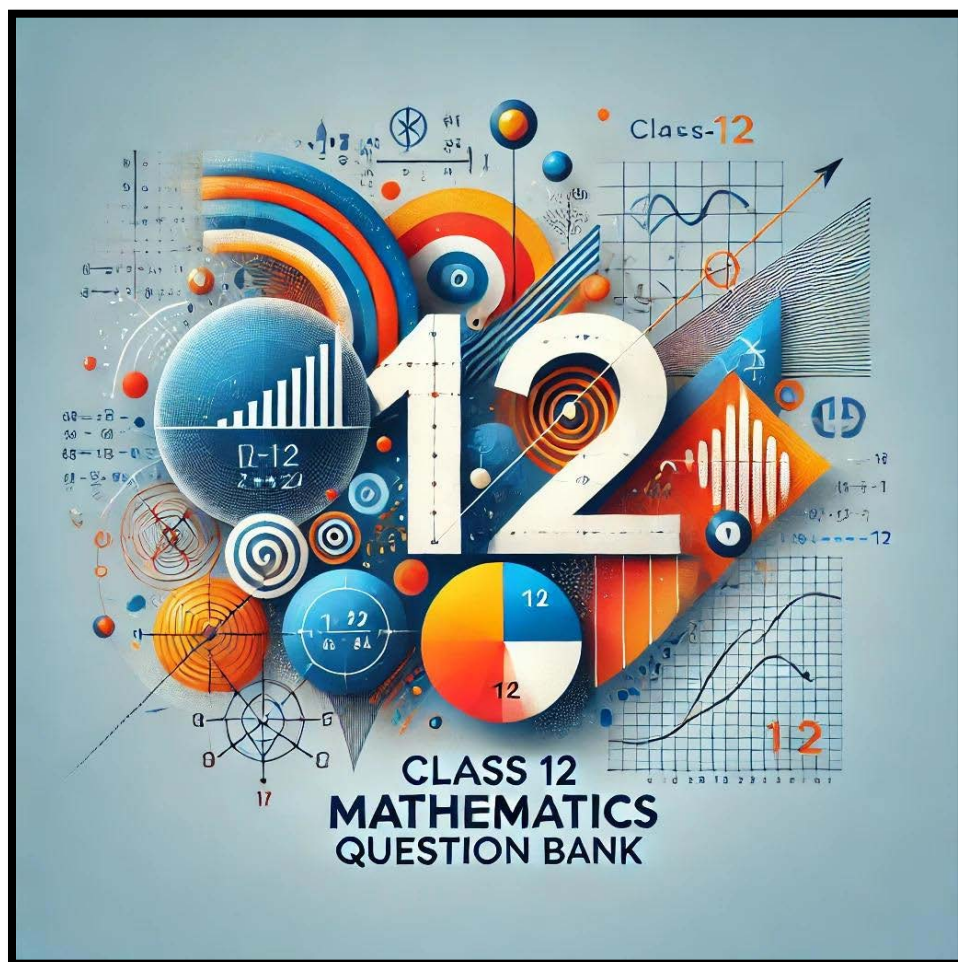
$$\Rightarrow a = 1 \quad \text{and} \quad b = -1$$

334 (d)

$$(AB^{-1}C)^{-1} = C^{-1}(B^{-1})^{-1}A^{-1} = C^{-1}BA^{-1}$$

1)	a	2)	d	3)	b	4)	b	205)	c	206)	a	207)	a	208)	a
5)	b	6)	c	7)	a	8)	b	209)	a	210)	a	211)	c	212)	c
9)	c	10)	d	11)	d	12)	d	213)	a	214)	b	215)	a	216)	a
13)	a	14)	d	15)	a	16)	b	217)	a	218)	b	219)	a	220)	c
17)	c	18)	a	19)	d	20)	b	221)	a	222)	d	223)	a	224)	b
21)	c	22)	b	23)	d	24)	d	225)	c	226)	d	227)	a	228)	c
25)	a	26)	b	27)	d	28)	b	229)	c	230)	c	231)	a	232)	a
29)	c	30)	a	31)	b	32)	d	233)	a	234)	d	235)	c	236)	c
33)	c	34)	a	35)	a	36)	a	237)	c	238)	d	239)	a	240)	d
37)	c	38)	c	39)	d	40)	c	241)	b	242)	b	243)	b	244)	c
41)	a	42)	a	43)	d	44)	a	245)	c	246)	a	247)	b	248)	d
45)	c	46)	d	47)	c	48)	c	249)	b	250)	b	251)	b	252)	d
49)	a	50)	a	51)	d	52)	a	253)	a	254)	d	255)	d	256)	b
53)	c	54)	c	55)	d	56)	d	257)	a	258)	c	259)	b	260)	a
57)	c	58)	d	59)	a	60)	b	261)	c	262)	a	263)	a	264)	a
61)	a	62)	c	63)	a	64)	d	265)	c	266)	d	267)	d	268)	d
65)	d	66)	b	67)	b	68)	b	269)	b	270)	a	271)	c	272)	a
69)	b	70)	c	71)	b	72)	a	273)	d	274)	c	275)	c	276)	d
73)	b	74)	d	75)	d	76)	a	277)	a	278)	d	279)	b	280)	a
77)	c	78)	d	79)	d	80)	a	281)	b	282)	d	283)	d	284)	b
81)	b	82)	b	83)	a	84)	b	285)	a	286)	d	287)	a	288)	c
85)	a	86)	c	87)	b	88)	d	289)	c	290)	b	291)	c	292)	b
89)	a	90)	d	91)	c	92)	a	293)	b	294)	c	295)	c	296)	b
93)	d	94)	a	95)	d	96)	b	297)	b	298)	b	299)	c	300)	a
97)	b	98)	b	99)	d	100)	b	301)	a	302)	b	303)	d	304)	b
101)	b	102)	b	103)	b	104)	a	305)	a	306)	d	307)	a	308)	c
105)	b	106)	d	107)	b	108)	b	309)	b	310)	c	311)	c	312)	d
109)	c	110)	b	111)	b	112)	a	313)	c	314)	c	315)	a	316)	b
113)	a	114)	a	115)	d	116)	c	317)	a	318)	c	319)	a	320)	a
117)	d	118)	d	119)	b	120)	d	321)	b	322)	b	323)	b	324)	a
121)	a	122)	d	123)	a	124)	a	325)	a	326)	c	327)	c	328)	b
125)	c	126)	a	127)	d	128)	d	329)	b	330)	b	331)	a	332)	b
129)	b	130)	b	131)	a	132)	d	333)	a	334)	d	335)	b		
133)	d	134)	b	135)	d	136)	c								
137)	a	138)	a	139)	c	140)	b								
141)	b	142)	a	143)	b	144)	a								
145)	c	146)	a	147)	b	148)	a								
149)	a	150)	d	151)	c	152)	a								
153)	b	154)	d	155)	b	156)	a								
157)	a	158)	a	159)	a	160)	a								
161)	a	162)	d	163)	c	164)	c								
165)	a	166)	a	167)	c	168)	d								
169)	b	170)	d	171)	c	172)	b								
173)	c	174)	c	175)	a	176)	c								
177)	b	178)	a	179)	a	180)	b								
181)	d	182)	a	183)	a	184)	a								
185)	c	186)	c	187)	b	188)	a								
189)	d	190)	a	191)	b	192)	b								
193)	d	194)	b	195)	a	196)	d								
197)	b	198)	a	199)	b	200)	d								
201)	a	202)	a	203)	d	204)	b								

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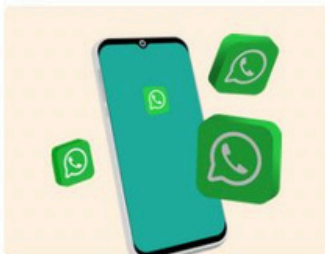
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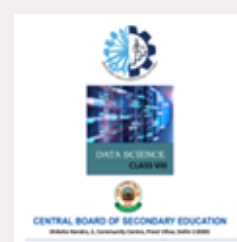
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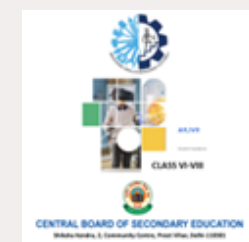
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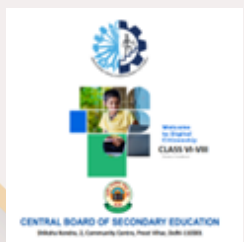
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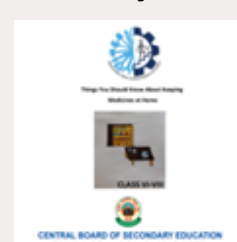
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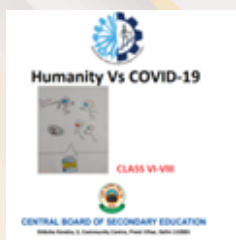
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Things you should know about keeping Medicines at home



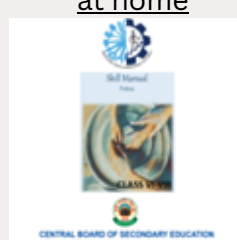
What to do when Doctor is not around



Humanity & Covid-19



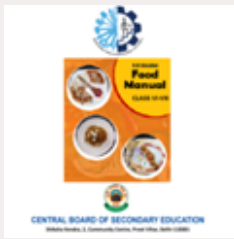
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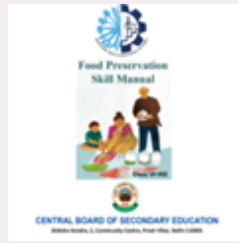
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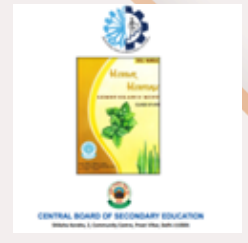
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Food Preservation



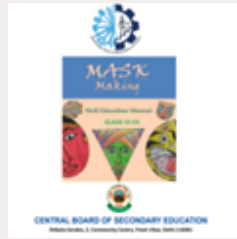
Baking



Herbal Heritage



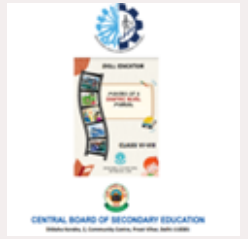
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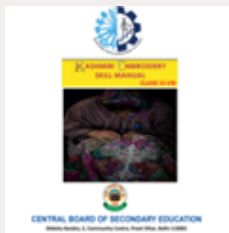
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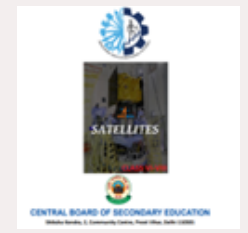
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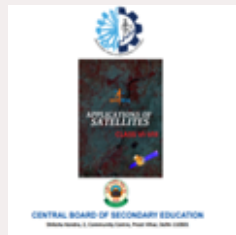
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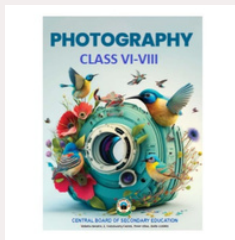
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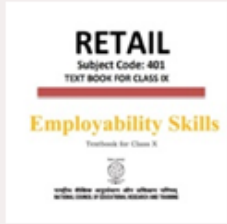


Application of Satellites

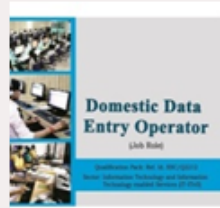


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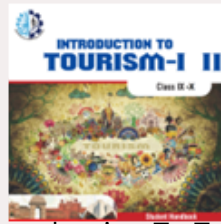
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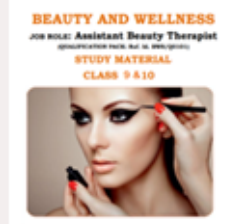
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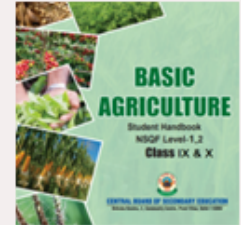
Introduction To Financial Markets



Introduction To Tourism



Beauty & Wellness



Agriculture



Food Production



Front Office Operations



Banking & Insurance



Marketing & Sales



Health Care



Apparel



Multi Media



Multi Skill Foundation Course



Artificial Intelligence



Physical Activity Trainer



Data Science



Electronics & Hardware (NEW)

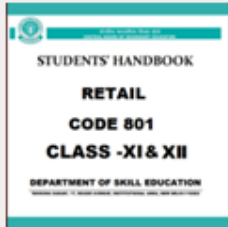


Foundation Skills For Sciences (Pharmaceutical & Biotechnology)(NEW)

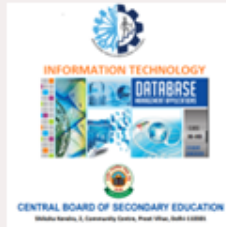


Design Thinking & Innovation (NEW)

SKILL SUBJECTS AT SR. SEC. LEVEL (CLASSES XI – XII)



Retail



Information Technology



Web Application



Automotive



Financial Markets Management



Tourism



Beauty & Wellness



Agriculture



Food Production



Front Office Operations



Banking



Marketing



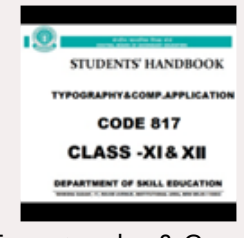
Health Care



Insurance



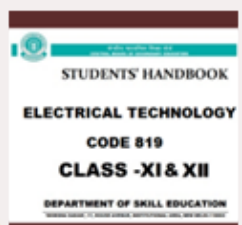
Horticulture



Typography & Comp.
Application



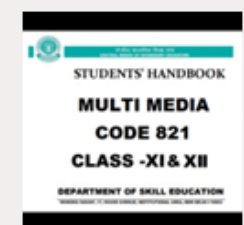
Geospatial Technology



Electrical Technology



Electronic Technology



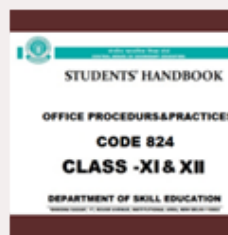
Multi-Media



Taxation



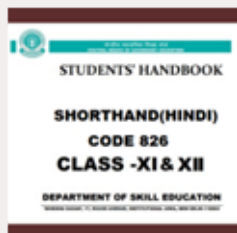
Cost Accounting



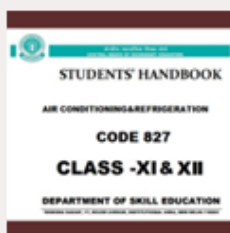
Office Procedures & Practices



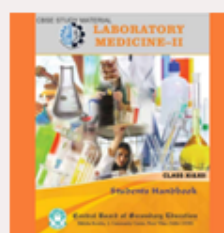
Shorthand (English)



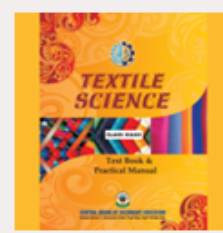
Shorthand (Hindi)



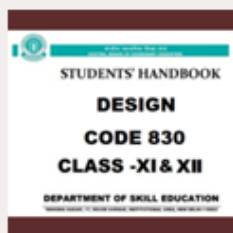
Air-Conditioning & Refrigeration



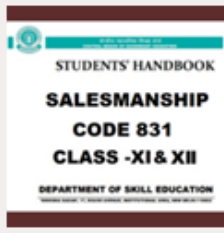
Medical Diagnostics



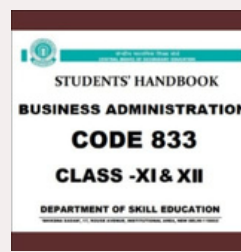
Textile Design



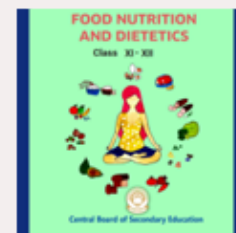
Design



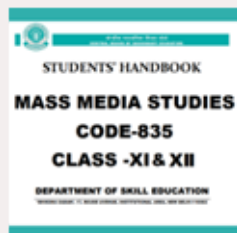
Salesmanship



Business Administration



Food Nutrition & Dietetics



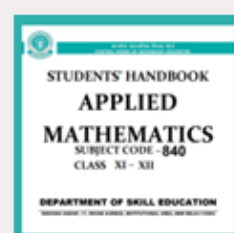
Mass Media Studies



Library & Information Science



Fashion Studies



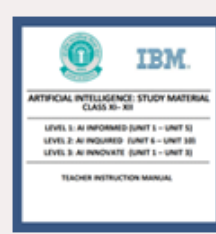
Applied Mathematics



Yoga



Early Childhood Care & Education



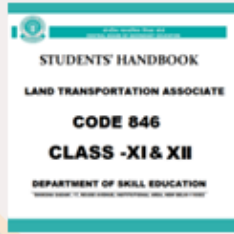
Artificial Intelligence



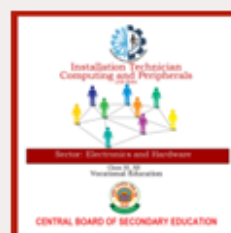
Data Science



Physical Activity Trainer(new)



Land Transportation Associate (NEW)



Electronics & Hardware (NEW)



Design Thinking & Innovation (NEW)

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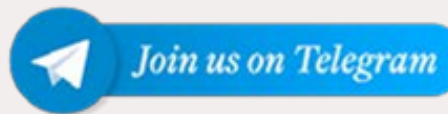
Artificial intelligence



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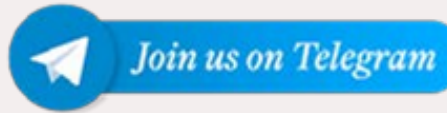
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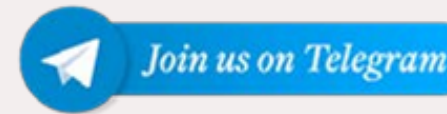
Class 1



Class 2



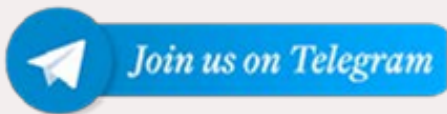
Class 3



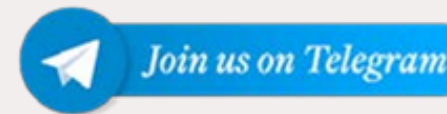
Class 4



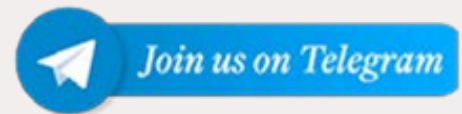
Class 5



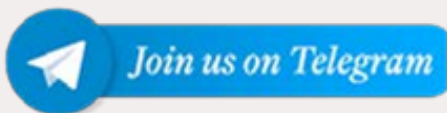
Class 6



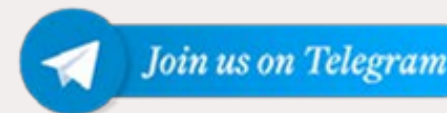
Class 7



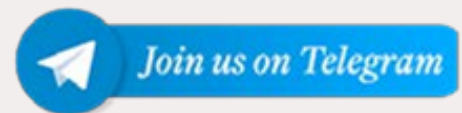
Class 8



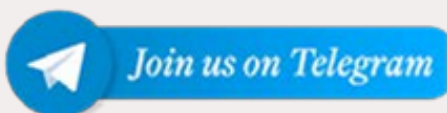
Class 9



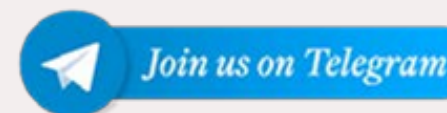
Class 10



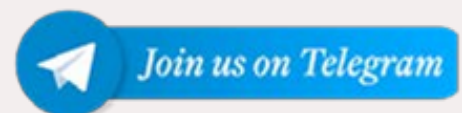
Class 11 (Sci)



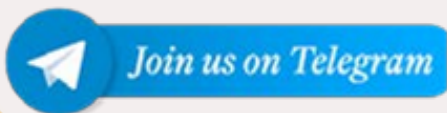
Class 11 (Com)



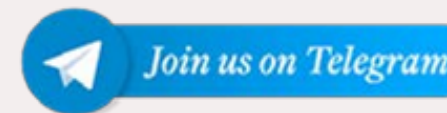
Class 11 (Hum)



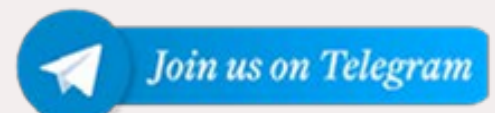
Class 12 (Sci)



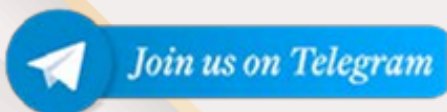
Class 12 (Com)



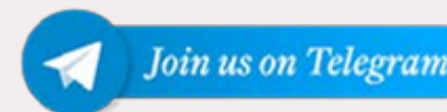
Class 12 (Hum)



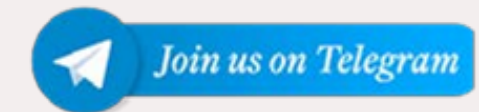
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